Exam #2, Math 205A (Linear Algebra)

This take-home exam is due at class time on **Friday, March 22**. (Sooner is fine.) You may consult the textbook (or any other book) and any class notes and handouts, but **please do not discuss any details of this exam with anyone except me!** Please sign the bottom of this sheet and turn it in with your exam. You may ask me questions about the exam, but I reserve the right to give unsatisfying answers. Matrix multiplications and reductions to reduced row echelon form may be done on MATLAB or a calculator, but please show all other work.

1. (16 points) Find the complete solution of the system $\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 3 & 5 & 1 \\ 3 & 4 & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 20 \end{pmatrix}$.

2. (22 points) Find a basis for each of the four subspaces associated with the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 & 2 & 3 \\ 2 & 3 & 7 & 3 & 4 \\ 1 & 2 & 4 & 1 & 1 \end{pmatrix}$$

What is the factored form of A that displays these bases?

3. (14 points) The vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 5 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix}$ span a 3-dimensional subspace of $\mathbb{R}^4$. Find an orthonormal basis for it.

4. (18 points) Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$, and $\vec{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 4 \\ 3 \\ 1 \end{pmatrix}$, and let S be the subspace of $\mathbb{R}^5$ spanned by $\vec{v}_1$ and $\vec{v}_2$. Find the matrix P that projects vectors in $\mathbb{R}^5$ onto S, and the matrix R that reflects through S. Find also the projection of $\vec{v}_3$ onto S and the reflection $\vec{r}$ of $\vec{v}_3$ through S.

5. (16 points) Explain how you can tell that $P = \frac{1}{20} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 3 & 11 & 9 & -3 \\ 3 & 9 & 9 & 3 \\ 1 & -3 & 3 & 19 \end{pmatrix}$ is a projection matrix. Find a basis for the subspace T of $\mathbb{R}^4$ that P projects onto, and a basis for $T^\perp$ (the orthogonal complement of T).

6. (8 points) Find the line which best fits the four data points $(1, 2)$, $(2, 1)$, $(3, 3)$ and $(4, 2)$ in the sense of least squares.

7. (6 points) Suppose U is a 6-dimensional subspace of $\mathbb{R}^8$, and let A be an 8×6 matrix whose columns are a basis of U. To find the projection matrix onto U directly from the formula $A\left(A^T A\right)^{-1} A^T$ one would have to invert a 6×6 matrix. Can you think of an alternative way to find this projection matrix where you would only need to invert a 2×2 matrix?

I affirm that I did not receive help from another person in doing this exam, nor did I give help to another student in the class.

(signed) ______________________________