Math 205A Quiz 08 page 1 March 21, 2008 NAME Suggested solution

1. Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 8 & -10 \\ 5 & 5 & -7 \end{bmatrix}$.

1A. Find the characteristic polynomial of A in factored form.

$$\text{char poly } A = \begin{vmatrix} 3-\lambda & 0 & 0 \\ 5 & 8-\lambda & -10 \\ 5 & 5 & -7-\lambda \end{vmatrix} = (3-\lambda)\begin{vmatrix} 8-\lambda & -10 \\ 5 & -7-\lambda \end{vmatrix} = (3-\lambda)\big((8-\lambda)(-7-\lambda)+50\big)$$

$$= (3-\lambda)(\lambda^2+7\lambda-8\lambda-56+50) = (3-\lambda)(\lambda^2-\lambda-6) = (3-\lambda)(\lambda-3)(\lambda-2)$$

$$= -(\lambda-3)^2(\lambda-2)$$

1B. One of the eigenvalues of A is an odd number, the other is even. Find a basis for the eigenspace of the odd one. Show all your work. $\lambda=3$ is odd. The eigenspace for $\lambda=3$ is the nullspace of:

$$(A-3I) = \begin{bmatrix} 3-3 & 0 & 0 \\ 5 & 8-3 & -10 \\ 5 & 5 & -7-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 5 & -10 \\ 5 & 5 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ soln's of $(A-3I)=\vec{0}$ are

$$\begin{cases} x_1 = -x_2+2x_3 \\ x_2 = \text{free} \\ x_3 = \text{free} \end{cases} \Rightarrow \vec{x} = x_2\begin{bmatrix}-1\\1\\0\end{bmatrix} + x_3\begin{bmatrix}2\\0\\1\end{bmatrix},$$

so a basis is $\left\{ \begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\0\\1\end{bmatrix} \right\}$

2. Let $M = \begin{bmatrix} 1 & -1 & -3 & 5 & 5 \\ 2 & 1 & 15 & 2 & 1 \\ 1 & 1 & 11 & 0 & -1 \\ -1 & 1 & 3 & -4 & -5 \end{bmatrix}$; then RREF($[M|I_4]$) is $\left[\begin{array}{ccccc|cccc} 1 & 0 & 4 & 0 & 2 & -2 & 0 & 1/2 & -5/2 \\ 0 & 1 & 7 & 0 & -3 & 2 & 0 & 1/2 & 5/2 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3/2 & 1/2 \end{array}\right]$

Find bases for each of the following; use correct notation. You may write "*same as*" if appropriate; for example, if the basis in (2G) were the same as for (2A) then "*same as 2A*" would be acceptable as the answer to 2G.

2A. Col(M) (use the pivot cols)

$$\left\{ \begin{bmatrix}1\\2\\1\\-1\end{bmatrix}, \begin{bmatrix}-1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}5\\2\\0\\-4\end{bmatrix} \right\}$$

these answers are <u>NOT</u> inter-changeable; make sure you know WHY!

2B. Nul(M) need soln's of $M\vec{x}=\vec{0}$; they are

$$\begin{cases} x_1 = -4x_3-2x_5 \\ x_2 = -7x_3+3x_5 \\ x_3 = \text{free} \\ x_4 = 0 \\ x_5 = \text{free} \end{cases} \Rightarrow \text{a basis is } \left\{ \begin{bmatrix}-4\\-7\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-2\\3\\0\\0\\1\end{bmatrix} \right\}$$

2C. Col(RREF(M))

$$\left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \right\}$$

2D. Nul(RREF(M))

Same as 2B (make sure you know WHY!)

2E. Row(M) use pivot rows

$$\left\{ \begin{bmatrix}1 & 0 & 4 & 0 & 2\end{bmatrix}, \begin{bmatrix}0 & 1 & 7 & 0 & -3\end{bmatrix}, \begin{bmatrix}0 & 0 & 0 & 1 & 0\end{bmatrix} \right\}$$

2F. Row(RREF(M))

same as 2E (99)