1. Suppose an economy is modeled with four sectors A, B, C, and D. Suppose that the entire output of D is consumed equally by the *other three* sectors, and the output of A is equally consumed by *all four* sectors. Suppose that B consumes none of its output, while half of it is used by D, 1/8 is used by A and the remainder goes to C. Finally, A uses none of C's output, B uses 1/8 of C's output, D uses twice what B does, and C consumes the rest.

1A. Find the exchange table for this economy. You may assume all columns sum to one.

1B. Find the complete set $\{P_A, P_B, P_C, P_D\}$ of equilibrium prices for this economy. Write down any system of equations and augmented matrices you use in solving this problem. (Note well: if you need to enter 1/3 into your calculator as a matrix entry, do it as "$1 \div 3$" rather than entering 0.333 or some such bad decimal approximation). Use fractions in your answers, not decimals.

1C. Suppose P_D is 100 dollars. Rank all four equilibrium prices from least to greatest.

2. Let $A = \begin{bmatrix} 2 & 7 & 10 & 10 & 4 \\ 1 & 3 & 5 & 2 & 2 \\ 2 & 11 & 11 & 35 & 7 \\ 3 & 10 & 16 & 13 & 9 \end{bmatrix}$; then the reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & -21 & -13 \\ 0 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Find a basis for each of the following:

a) Col(A)

b) Col(R)

c) Nul(A)

d) Nul(R)

e) What is the rank of A?

f) Explain why Col(A) cannot be the same as Col(R) here.

Now let $B = \begin{bmatrix} 2 & 7 & 10 & 10 & 4 \\ 1 & 3 & 5 & 2 & 2 \\ 2 & 11 & 11 & 35 & 7 \end{bmatrix}$; it's true that RREF($B$) is $S = \begin{bmatrix} 1 & 0 & 0 & -21 & -13 \\ 0 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & 1 & 3 \end{bmatrix}$

g) Explain why Col(B) and Col(S) *are* equal here, as they are both $\mathbb{R}^3$. (Think about whether you can always solve $M\mathbf{x} = \mathbf{b}$ for any $\mathbf{b} \in \mathbb{R}^3$, where M is either matrix).

h) In this problem, is a basis for Col(B) also one for Col(S) and *vice versa*?

3. Let $A = \begin{bmatrix} 2 & 7 & -2 \\ 0 & 5 & 0 \\ -3 & 7 & 3 \end{bmatrix}$.

a) Find the characteristic polynomial of A; show all your steps.

b) List the eigenvalues of A and their multiplicities:

c) Find a basis for each eigenspace.

d) Here, A diagonalizable. Show this by giving P, D and P^{-1}.

Let $B = \begin{bmatrix} 2 & 2 & 3 \\ -5 & 5 & 5 \\ 2 & 2 & 3 \end{bmatrix}$; then B has the same eigenvalues and multiplicities as A (You do not need to check this). Yet B is not diagonalizable. So what must "go wrong"? Verify this by exhibiting a basis for the right eigenvalue.

4. Let H be the subset of $\mathbb{R}^3$ consisting of all vectors of the form $\mathbf{a} = \begin{bmatrix} a^2 \\ a \\ a^3 \end{bmatrix}$, where a can be any real number.

Is H a subspace of $\mathbb{R}^3$? For each of the three parts of the definition of a subspace, state that part of the definition and then prove H passes that part, or give a counterexample to show why it fails that part.

i)

ii)

iii)

5. Let $M = \begin{bmatrix} 4 & a & b \\ 0 & 1 & c \\ 0 & 0 & 5 \end{bmatrix}$. What is the determinant of M? ☐

Now, find the determinants of each of the following matrices and write your answers in the boxes.

3M ☐

M^3 ☐

$\begin{bmatrix} 0 & 0 & 5 \\ 8 & 1+2a & c+2b \\ 0.4 & a/10 & b/10 \end{bmatrix}$ ☐

$\begin{bmatrix} 4 & a & b \\ 0 & 1 & c \\ 8 & 2a & 2b \end{bmatrix}$ ☐

$2M + 3I_3$ ☐

M^{-1} ☐

M^T ☐

$(M^T)^{-1}$ ☐