

Math 205A Test 2 (50 points)

Name: _____

- Check that you have 7 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (6 points) Short answers: (Show all the calculations to get the answers. No explanations needed.)

(a) For a 2×2 matrix B , $\det B = -8$. Find $\det 3B$.

(b) Let $\vec{u}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and let $\vec{u}_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. Then the set $\mathcal{B} = \{\vec{u}_1, \vec{u}_2\}$ is a basis for \mathbb{R}^2 . Find \vec{y} such that $[\vec{y}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$.

(c) Suppose 0 is an eigenvalue of a 3×5 matrix A and the dimension of the corresponding eigenspace is 2. Find $\text{rank } A$.

2. (6 points) Let $A = \begin{bmatrix} 2 & 0 & 3 \\ 6 & 2 & 8 \\ 0 & 4 & -2 \end{bmatrix}$. Find a basis for $\text{Col } A$ and then state the dimension of $\text{Col } A$.

3. (7 points) Let $A = \begin{bmatrix} 10 & -4 \\ 9 & -2 \end{bmatrix}$. The only eigenvalue of A is 4. Is A diagonalizable? If so, find the matrices P and D so that $A = PDP^{-1}$. If not, explain why not.

4. (7 points) Let A be a 3×3 matrix. Suppose \vec{u} is an eigenvector of A corresponding to the eigenvalue 7.

(a) Can you say anything about the value of $\det(A - 7I)$? Why?

(b) Can you say anything about the value of $\det(A - 10I)$? Why?

(c) Is \vec{u} an eigenvector of $2I - A$? If so, find the corresponding eigenvalue. If not, explain why not.

5. (8 points) Let $\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$. Let $L = \text{Span}\{\vec{u}\}$.

(a) Compute the distance from \vec{y} to L .

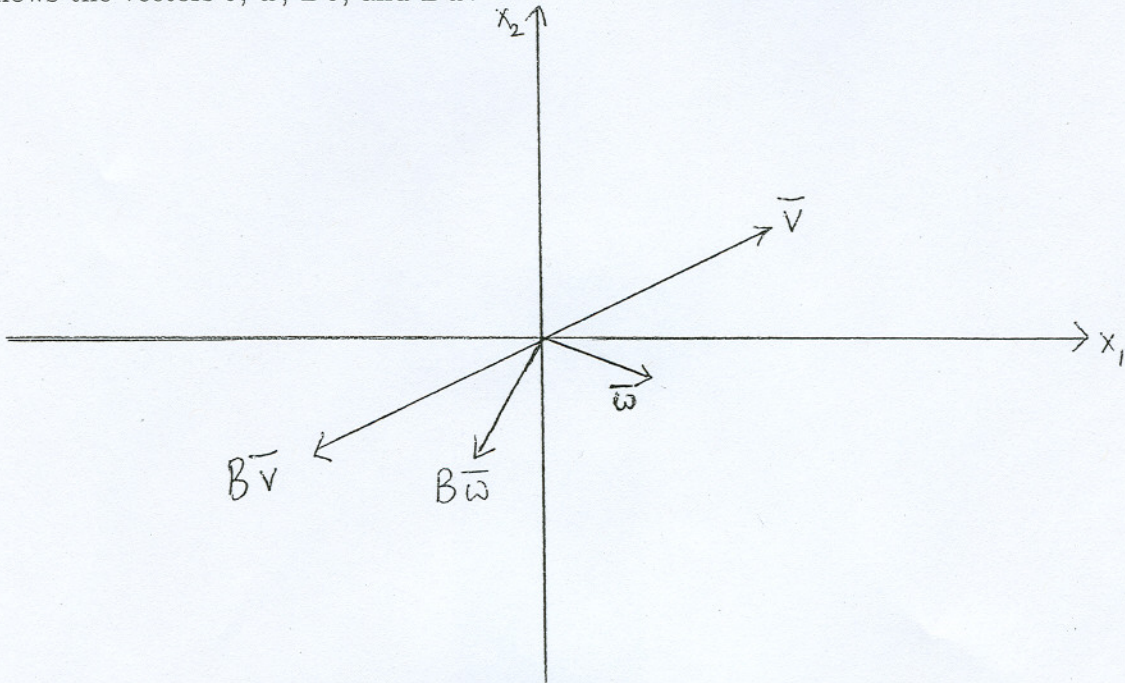
(b) Find a vector in L^\perp .

6. (8 points) Let $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be an orthogonal set in \mathbb{R}^4 .

(a) Is the set $\{-5\vec{v}_1, 2\vec{v}_2, \vec{v}_3\}$ a linearly independent set? Explain.

(b) Is the set $\{-5\vec{v}_1, 2\vec{v}_2, \vec{v}_3\}$ a basis for \mathbb{R}^4 ? Explain.

7. (8 points) Suppose B is a 2×2 matrix and \vec{v} and \vec{w} are two vectors in \mathbb{R}^2 . The following figure shows the vectors \vec{v} , \vec{w} , $B\vec{v}$, and $B\vec{w}$.



- (a) Is \vec{v} an eigenvector of B ? If so, what is the corresponding eigenvalue? If not, explain why not.
- (b) Is \vec{w} an eigenvector of B ? If so, what is the corresponding eigenvalue? If not, explain why not.
- (c) Draw the orthogonal projection of \vec{v} onto \vec{w} . Show how you draw the projection vector and indicate the projection vector clearly in the above figure.