MATH 205 EXAM 2 March 20, 2008

Name: ____________________

Your grade is based on correctness, completeness, and clarity on each exercise. You may use a calculator, but no notes, books, or other students. Good luck!

1.) (15 pts.)

a.) (10 pts.) Consider the subset of $\mathbb{P}_2$ of all polynomials of the form $\mathbf{p}(t) = a + bt^2$, where a is in $\mathbb{R}$ and b is in $\mathbb{R}$. Demonstrate that this subset is a subspace of $\mathbb{P}_2$.

b.) (5 pts.) Let H be the set of points inside and on a circle of radius 2 that is centered at the origin of the $xy-$plane. That is, $H = \{(x, y) : x^2 + y^2 \leq 4\}$. Use an example (two vectors, or a vector and a scalar) to show that H is *not* a subspace of $\mathbb{R}^2$.

2.) (15 pts.) Use the Invertible Matrix Theorem (IMT) to respond to the following questions. Be sure to state clearly which parts of the IMT you are using. In all cases, assume the matrix A is $n \times n$.

a.) (5 pts.) If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then does A have fewer than n pivot positions?

b.) (5 pts.) If 0 is *not* an eigenvalue of A, is A^T invertible?

c.) (5 pts.) Suppose the columns of A form a basis for $\mathbb{R}^n$. What can you say about dim Col A? What can you say about dim Nul A?

3.) (15 pts.)

a.) (5 pts.) Consider the set $\left\{ \begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} : a,b,c,d \text{ real} \right\}$. Find a matrix A, having linearly independent columns, for which this set is Col A.

b.) (5 pts.) For the matrix $B = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, find a basis for Nul B.

c.) (5 pts.) For the matrix B above, what is dim Nul B? What is rank B?

4.) (15 pts.)

a.) (7 pts.) Is $\lambda = 2$ an eigenvalue of $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$? Why or why not?

b.) (8 pts.) Is $\mathbf{u} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ an eigenvector of $B = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$? Why or why not?

5.) (15 pts.) Use any theorems or other results from this semester to respond to the following questions. Be sure to state clearly which theorems, parts of theorems, or other reasoning you are using.

a.) (5 pts.) Let A be an $n \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then is there a *unique* solution for each $\mathbf{b}$ in $\mathbb{R}^n$?

b.) (5 pts.) If $n \times n$ matrices E and F have the property that $EF = I$, then must $FE = I$?

c.) (5 pts.) Let B be an $m \times n$ matrix, where m and n are not necessarily equal. Is it still true that dim Row $B =$ dim Col B?

6.) (15 pts.) Diagonalize the matrix $A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$. *Hint: be clever when computing the eigenvalues.*

7.) (10 pts.) Let $A = \begin{bmatrix} -2 & 3 & 1 \\ 0 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix}$. Complete the following tasks *by hand*, showing appropriate work. Use the back of this page if you need more room.

a.) (5 pts.) Compute AB for $B = \begin{bmatrix} 4 & -3 \\ 5 & -2 \\ 2 & -1 \end{bmatrix}$.

b.) (5 pts.) Use the algorithm discussed in Chapter 2 to compute A^{-1}.