

1. Let $A = \begin{bmatrix} 3 & -8 & 14 & -1 \\ 1 & 1 & 1 & 3 \\ 2 & 0 & 4 & 1 \\ 1 & 2 & 0 & 2 \end{bmatrix}$; then $[A \mid I_4]$ is row-equivalent to $\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 0 & -2/7 & 4/7 & 1/7 \\ 0 & 1 & -1 & 0 & 0 & -3/7 & -1/7 & 5/7 \\ 0 & 0 & 0 & 1 & 0 & 4/7 & -1/7 & -2/7 \\ 0 & 0 & 0 & 0 & 1 & -2 & -3 & 5 \end{array} \right]$.

Let $R = \text{rref}(A)$.

1A. Find a basis for $\text{Col}(A)$.

1B. Find a basis for $\text{Col}(R)$.

1C. Is 0 an eigenvalue of A ? If so, find a basis for the eigenspace of 0, if 0 is not an eigenvalue, explain why not. (You do *not* need to find any polynomials in this problem!)

This is a continuation of problem 1. For your reference we copy the following:

$$A = \begin{bmatrix} 3 & -8 & 14 & -1 \\ 1 & 1 & 1 & 3 \\ 2 & 0 & 4 & 1 \\ 1 & 2 & 0 & 2 \end{bmatrix}; \text{ and } [A | I_4] \text{ is row-equivalent to } \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 0 & -2/7 & 4/7 & 1/7 \\ 0 & 1 & -1 & 0 & 0 & -3/7 & -1/7 & 5/7 \\ 0 & 0 & 0 & 1 & 0 & 4/7 & -1/7 & -2/7 \\ 0 & 0 & 0 & 0 & 1 & -2 & -3 & 5 \end{array} \right].$$

1D. Find all conditions both necessary and sufficient for a vector \mathbf{b} with entries b_1, \dots, b_4 to be in $\text{Col}(A)$.

1E. Find all conditions both necessary and sufficient for a vector \mathbf{b} with entries b_1, \dots, b_4 to be in $\text{Col}(R)$.

1F. Find a non-zero vector which is in $\text{Col}(A)$ but not $\text{Col}(R)$. Show your reasoning.

1G. Find a non-zero vector which is in $\text{Col}(R)$ but not $\text{Col}(A)$. Show your reasoning.

1H. Find a non-zero vector which is in both $\text{Col}(A)$ and $\text{Col}(R)$. Show your reasoning.

2. Suppose an economy is modeled with four sectors A , B , C , and D . Suppose that sector C consumes $1/4$ of every sector's output, including that of C itself. Suppose that the output of A is equally divided by and consumed by all four sectors. Suppose that D consumes half of its own output, while B uses none of its own, and none of C 's. What C and D do not consume of D 's output is divided equally and used by A and B ; B and D consume equal amounts of each other's output and this goes for C and D as well. Finally, the output of B not used by sectors B , C , and D is consumed by A and A also consumes any output of C not used by sectors B , C , and D .

2A. Find the exchange table for this economy. You may assume all columns sum to one.

2B. Find the complete set $\{P_A, P_B, P_C, P_D\}$ of equilibrium prices for this economy. Write down any system of equations and augmented matrices you use in solving this problem. (Note well: if you need to enter $1/3$ into your calculator as a matrix entry, do it as " $1 \div 3$ " rather than entering 0.333 or some such bad decimal approximation). Use fractions in your answers, not decimals.

2C. Suppose P_D is 100 dollars. Rank all four equilibrium prices from least to greatest.

3. Let $A = \begin{bmatrix} -9 & 24 & 12 \\ -4 & 11 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

3A. Find the characteristic polynomial of A . Show all your work. Choose your “expansion” wisely and don’t make any sign mistakes at all. Show all your work. Write your polynomial in *completely factored* form.

3B. What are the eigenvalues of A , and their multiplicities?

3C. Find a basis for the eigenspaces of each of those eigenvalues. Organize your work nicely; make sure it’s clear which basis goes with which eigenvector.

3D. Is A diagonalizable? If so find P , D and P^{-1} with the appropriate properties; if A is not diagonalizable, explain why it’s not.

4. Let $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 5 & 0 \\ -4 & -4 & -1 \end{bmatrix}$

4A. Show that B has the same characteristic polynomial as does the matrix A in problem 3. Show your work, make no sign errors, and again, write your polynomial in *completely factored* form.

4B. This matrix B is not diagonalizable. Given this, what *must* be the case about one of the eigenvalues and the dimension of its eigenspace? (In your answer, say which eigenvalue has to be “at fault”).

4C. Verify your answer to 4B by finding that eigenspace.

4D. Even though B is not diagonalizable, you should still be able to find a matrix Q such that $BQ = QD$ where D is the same as in problem 3. Do it.

5. Suppose that when applied in the order: “first do Op1, then do Op2”, the following row operations turn the matrix A into the matrix $B = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$:

Op1: three copies of row 2 are subtracted from row 1.

Op2: row 2 is multiplied by five.

5A: What elementary matrices F and G represent the operations Op1 and Op2, respectively?

5B: Which matrix equation is true? (1) $FGA = B$ (2) $GFA = B$

5C. Find $\det(A)$.

5D. Find A .

5E. Find A^{-1} .

6. Suppose $B = \begin{bmatrix} x & y & z \\ 3 & 4 & 6 \\ a & b & c \end{bmatrix}$ has determinant 10.

Find the determinant of each of the following matrices. You do *NOT* need to list any rules about matrices you used to find the det. (eg, “swapping rows changes the sign of the det” or “the inverse of the derivative of a matrix is the matrix of its eigenzeigen” (this second fact is nonsense). Just FIND the determinants.

6A. $\begin{bmatrix} x-18 & y-24 & z-36 \\ 3 & 4 & 6 \\ a & b & c \end{bmatrix}$ the det is:

6B. $\begin{bmatrix} -18 & -24 & -36 \\ 3 & 4 & 6 \\ a & b & c \end{bmatrix}$ the det is:

6C. $\begin{bmatrix} x & y & z \\ 1/4 & 1/3 & 1/2 \\ a & b & c \end{bmatrix}$ the det is:

6D. $\begin{bmatrix} x & 3 & a \\ y & 4 & b \\ z & 6 & c \end{bmatrix}$ the det is:

6E. $4B$ the det is:

6F. B^{-1}

BONUS: At the review we mentioned that the intersection of two subspaces of a vector space is another subspace. Find a basis for $\text{Col}(A) \cap \text{Col}(R)$ for A and R from problem 1. Use the back of this sheet if necessary.