

Math 205A Winter 10
Test 2 (50 points)

Name: _____

- Check that you have 6 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (6 points) Let $C = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 4 & 1 & -5 & 2 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 2 \end{bmatrix}$. Find all the eigenvalues of C .

2. (6 points) Suppose 0 is an eigenvalue of a 6×6 matrix A and \vec{u} is an eigenvector of A corresponding to the eigenvalue 0.

(a) Is A invertible? Explain.

(b) Is \vec{u} an eigenvector of A^2 ? If so, find the corresponding eigenvalue. If not, explain why not.

3. (12 points) Let $B = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{bmatrix}$. The eigenvalues of B are 2 and 4.

(a) Find a basis for the eigenspace corresponding to each eigenvalue of B .

(b) Is $\vec{x} = \begin{bmatrix} 4 \\ 2 \\ 7 \end{bmatrix}$ an eigenvector of B ? If so, find the coordinates of \vec{x} with respect to the basis (you found in part (a)) of the eigenspace to which it belongs. If not, explain why not.

(c) Is B diagonalizable? If so, find the matrices P and D so that $B = PDP^{-1}$. If not, explain why not.

4. (10 points) Let $W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} \right\}$.

(a) Find a basis and the dimension of W .

(b) Find a basis and the dimension of W^\perp (the orthogonal complement of W).

(c) Give a geometric description of W and W^\perp .

5. (8 points) Let A be a 2×2 matrix and B be a 2×5 matrix, with $\det A = -4$ and $\det B = 7$.

(a) Find $\det 3B$.

(b) Find $\det A^3$. Is A^3 an invertible matrix? Explain.

(c) What is the largest possible rank of B ? What is the smallest possible dimension of $\text{Nul } B$? Provide explanations for your answers.

6. (8 points) Let $\vec{v} = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ and let $\vec{w} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix}$.

(a) Find the distance between \vec{v} and \vec{w} .

(b) Compute $(\vec{v} \cdot \vec{w}) \vec{v}$.

(c) Find a vector of length 5 in the direction of \vec{w} .