

Math 205 (Winter 2016)

Test 2 (50 points)

Name: _____

- Check that you have eight questions on three pages.
- Show all your work to receive full credit for a problem. Points will be taken off if you do not show how you arrived at your answer, even if the answer is correct.
- Please keep your explanations brief; be clear and to the point. Points will be taken off for incorrect or irrelevant statements.

1. (6 points) Let A and B be 6×6 matrices, with $\det A = -10$ and $\det B = 5$. Use properties of determinants to compute:

(a) $\det 3A$

(b) $\det (A^T B^{-1})$

2. (7 points) Let $A = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$. This matrix A has only one eigenvalue which is 5.

(a) Find a basis for the eigenspace corresponding to the eigenvalue 5.

(b) Is the matrix A diagonalizable? Explain.

3. (9 points) Let $\vec{v} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$.

(a) Find a unit vector in the direction of \vec{v} .

(b) Find the distance between \vec{v} and \vec{y} .

(c) Let $L = \text{Span}\{\vec{v}\}$. Compute the orthogonal projection of \vec{y} onto L .

4. (4 points) Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is an orthogonal set of non-zero vectors in \mathbb{R}^4 . Is the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ a basis for \mathbb{R}^4 ? Explain.

5. (6 points) Suppose $\{\vec{u}, \vec{v}\}$ is a basis of the eigenspace corresponding to the eigenvalue 0 of a 5×5 matrix A .

(a) Is $\vec{w} = \vec{u} - 2\vec{v}$ an eigenvector of A ? If so, find the corresponding eigenvalue. If not, explain why.

(b) Find the dimension of $\text{Col } A$.

6. (8 points) Let $C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -3 \\ 1 & 2 & 1 \end{bmatrix}$.

(a) Find a basis for $\text{Col } C$.

(b) Let $\vec{x} = \begin{bmatrix} -3 \\ -11 \\ 7 \end{bmatrix}$ be a vector in $\text{Col } C$. Find the coordinates of \vec{x} with respect to the basis you found in part (a).

7. (5 points) Let $B = \begin{bmatrix} -2 & 0 & -9 \\ 0 & 3 & 0 \\ 0 & 5 & 6 \end{bmatrix}$. Find all the eigenvalues of B .

8. (5 points) Let $W = \text{Span} \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$. Is $\begin{bmatrix} 2 \\ 6 \\ 4 \\ 0 \end{bmatrix}$ in W^\perp ? Explain.