MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

EXAM II - MARCH 18, 2013

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. Let

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}.$$

(a)(7 pts) Find the eigenvalues of A.

(b)(7 pts) For each of the eigenvalue found in (a), determine the corresponding eigenspaces by giving a basis for each such subspace.

(c)(6 pts) Is A diagonalizable? If so, find an invertible matrix P such that $P^{-1}AP$ is diagonal.

2. Let

$$A = \begin{bmatrix} 1 & 4 & 3 & -1 \\ 0 & 1 & 2 & -1 \\ 2 & 5 & 0 & 0 \end{bmatrix}.$$

(a)(8 pts) Find a basis for the column space Col$A$ of A.

(b)(8 pts) Find a basis for the null space Nul$A$ of A.

(c)(4 pts) What is the rank of A? Justify your answer.

3. (a)(5 pts) Let A be a 4×3 matrix. If $\dim \text{Nul} A = 3$, what is the rank of A? Justify your answer.

(b)(5 pts) Suppose B is a 5×5 matrix with eigenvalues $2, -1, 4$ such that the eigenspace corresponding to 2 has dimension 2; the eigenspace corresponding to -1 has dimension 1; and the eigenspace corresponding to 4 has dimension 1. Determine whether B is diagonalizable. Justify your answer.

(c)(5 pts) Let B be the matrix as in (b). Find $\det(B + I)$, the determinant of the matrix $(B + I)$. Justify your answer.

(d)(5 pts) Let B be the matrix as in (b). What is the dimension of $\text{Col}(B - 2I)$? Justify your answer.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by

$$T(x_1, x_2) = (x_1 + x_2, -4x_1 - 3x_2).$$

Consider the basis $\mathcal{B} = \{\begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix}\}$.

(a)(8 pts) Find the $\mathcal{B}$-matrix of the transformation T.

(b)(6 pts) Suppose $\vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find $[T(\vec{x})]_{\mathcal{B}}$, the $\mathcal{B}$-coordinates of $T(\vec{x})$.

(c)(6 pts) If $\mathcal{C} = \{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$ is another basis, find $[T(\vec{x})]_{\mathcal{C}}$.

5. (a)(7 pts) Find a basis for the subspace H of $\mathbb{R}^3$ spanned by the vectors $\begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 10 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \end{bmatrix}$.

(b)(7 pts) Let $T : \mathbb{P}_2 \to \mathbb{R}^2$ be the linear transformation given by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(0) \end{bmatrix}$. Find polynomials $\mathbf{p}_1, \mathbf{p}_2$ in $\mathbb{P}_2$ that span the kernel of T.

(c)(6 pts) Let λ be an eigenvalue of an $n \times n$ invertible matrix A and $\lambda \neq 0$. Show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}. [Hint: Let $\vec{v}$ be an eigenvector of A corresponding to λ.]