1. Suppose at the point \((x, y, z)\) in \(\mathbb{R}^3\) the temperature \(T\) in degrees F is given by \(T(x, y, z) = xy^2 + z^3\), where \(x, y\) and \(z\) are measured in feet.

1A) Let \(P\) be the point \((2, 1, 3)\). Find the equation of the plane tangent to the level surface of \(T\) at \(P\).

1B) What is the value of \(T\) on this level surface?

1C) At the point \(P\), in what direction is the change in temperature maximized?
2. Let \( T(x, y, z) = xy^2 + z^3 \) be the same as in problem (1). Now suppose that an object moves through \( \mathbb{R}^3 \) with position at time \( t \) is given by \( s(t) = (s_1(t), s_2(t), s_3(t)) \), where \( t \) is in minutes, but the component functions \( s_1, s_2 \) and \( s_3 \) are not given explicitly. Suppose at \( t = 2 \) the object is at \( P = (2, 1, 3) \).

2A) Note that the composition \( g = T \circ s \) represents the temperature of the object at time \( t \). Find the formula for \( dg/dt \); express your answer in terms of \( t \) (that is, in terms of \( s_1, s_2 \) and \( s_3 \) and their derivatives).

2B) Suppose the Jacobian matrix of \( s \) at \( t = 2 \) is \( Js \big|_{t=2} = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \); note this is just the velocity vector for the object at \( t = 2 \).

What is the directional derivative of \( T \) at \( P \) in the direction of this velocity vector?

2C) In degrees per minute, what is the rate of change in temperature experienced by the object at time \( t = 2 \)?
3. Suppose \( u = (xy^3, x^2y^2) \) and \( v = (v_1(r, s, t), v_2(r, s, t)) \). Let \( F = u \circ v \). In terms of \( r, s \) and \( t \), find and simplify as much as possible, \( \partial F_2 / \partial t \).
4A. Use Maple to find the first, second and third order Taylor polynomials for \( f(x, y) = \frac{32xy}{x + y^2 - 10} \) for the point \( a = (2, 4) \), that is, in powers of \((x - 2)\) and \((y - 4)\). Use of Maple is restricted to finding the required partial derivatives and evaluating them at (2,4). PRINT your Maple work when your are done at the computer.

4B. Use your second order Taylor polynomial to approximate \( f(2.1, 4.1) \). Compare the approximation to the actual value by finding the difference. Use five decimal places in your final answers.
5. Let $f(x, y) = x + y + \sin(x^2 + y^2)$; then we have the following partial derivatives:

\[
\begin{align*}
    f_x &= 1 + 2 \cos(x^2 + y^2) \cdot x \\
    f_y &= 1 + 2 \cos(x^2 + y^2) \cdot y \\
    f_{xx} &= -4 \sin(x^2 + y^2) \cdot x^2 + 2 \cos(x^2 + y^2) \\
    f_{xy} &= -4 \sin(x^2 + y^2) \cdot y \\
    f_{yy} &= -4 \sin(x^2 + y^2) \cdot y^2 + 2 \cos(x^2 + y^2)
\end{align*}
\]

5a) A plot of level curves for this function is on the next page. It implies a critical point exists at $(1.47770, 1.47770)$ (to 5 decimal places). Show this point also (very very closely) satisfies the appropriate test to be a critical point. [NOTE WELL: this CP is not necessarily the one you see closest to the origin!]

5b) Use the second derivative test at $(1.47770, 1.47770)$ to see if it’s a local max, min, or saddle point. Show the values of any partial derivatives and other relevant expressions you use; list them all to 5 decimal places.

5c) There seems to be another critical point near $(2.01308, 2.01308)$ (to 5 decimal places). Show this point (very very closely) satisfies the appropriate test to be a critical point.

5d) Use the second derivative test at $(2.01308, 2.01308)$ to see if it’s a local max, min, or saddle point. Show the values of any partial derivatives and other relevant expressions you use; list them all to 5 decimal places.
6. Suppose that $f : \mathbb{R}^3 \to \mathbb{R}$. In each case below, the Hessian matrix given was found at one of the critical points of $f$. Use the second derivative test, if possible, to decide if the critical point is a local max, min, saddle point.

6A: 
\[
\begin{bmatrix}
5 & 2 & -2 \\
2 & 2 & 3 \\
-2 & 3 & 20
\end{bmatrix}
\]

6B: 
\[
\begin{bmatrix}
-5 & 2 & -2 \\
2 & 2 & 3 \\
-2 & 3 & 20
\end{bmatrix}
\]

6C: 
\[
\begin{bmatrix}
1 & 2 & -2 \\
2 & 4 & 3 \\
-2 & 3 & 20
\end{bmatrix}
\]