Math 205 (Winter 2011)

Test 2 (50 points)

Name: ______________________________

- Check that you have 8 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (5 points) Let A and B be 3×3 matrices, with $\det A = 4$ and $\det A^2B^{-1} = -8$. Use properties of determinants to compute:

   (a) $\det 2A$

   (b) $\det B$

2. (4 points) Let $H = \text{Span}\,\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_6\}$ be a subspace of $\mathbb{R}^5$. Is the set $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_6\}$ a basis for H? Explain.

3. (6 points) Suppose $\vec{u}$ is an eigenvector of a 4×4 matrix A corresponding to the eigenvalue -3.

   (a) Is $A + 3I$ an invertible matrix? Explain.

   (b) Show that $\vec{u}$ is an eigenvector of A^3 and find the corresponding eigenvalue.

4. (4 points) Determine if the following set is a subspace of the appropriate space. If the set is a subspace, find a basis and the dimension of the subspace. If the set is not a subspace, provide a counterexample to illustrate that one of the conditions in the definition of subspace does not hold.

$$W = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \text{ where a, b are non-negative real numbers} \right\}.$$

5. (8 points) Let $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and let $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$.

(a) Compute $\left(\dfrac{\vec{v} \cdot \vec{w}}{\vec{v} \cdot \vec{v}}\right) \vec{v}$ and call it $\vec{y}$.

(b) Compute $\vec{w} - \vec{y}$ and call it $\vec{z}$.

(c) Let $L = \text{Span}\{\vec{v}\}$. Which of the two vectors $\vec{y}$ and $\vec{z}$ is in $L^{\perp}$? Explain.

(d) Find the distance between $\vec{v}$ and $\vec{w}$.

6. (9 points) Suppose a 6×6 matrix A has only three distinct eigenvalues, 1, 0 and -1. Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis for the eigenspace corresponding to the eigenvalue 1 and $\{\vec{w}_1, \vec{w}_2\}$ is a basis for the eigenspace corresponding to the eigenvalue 0.

(a) Is A diagonalizable? Explain.

(b) Let $\vec{b} = 2\vec{v}_1 - 5\vec{v}_2$ be a vector in $\mathbb{R}^6$. Is $\vec{b}$ an eigenvector of A? Explain. If it is an eigenvector, find the corresponding eigenvalue.

(c) What is dim Nul A and rank A? Explain.

7. (7 points) Let $B = \begin{bmatrix} 1 & 3 & 1 & 2 & -4 \\ 0 & -1 & 5 & -1 & 6 \\ 2 & 5 & 0 & 3 & -9 \\ 0 & 2 & 3 & 2 & 1 \end{bmatrix}$.

(a) Find a basis for Col B and then state the dimension of Col B.

(b) Let $\vec{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 2 \end{bmatrix}$. Find the coordinates of $\vec{x}$ with respect to the basis you found in part (a).

8. (7 points) Let $C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$.

(a) Find all the eigenvalues of C.

(b) Find a basis for Nul C.