1. Let \( C = \begin{bmatrix} 10 & 5 & -10 & 65 & 0 & 30 \\ 4 & -7 & 14 & -1 & 0 & -42 \\ -5 & 0 & 0 & -25 & 1 & -2 \\ -8 & 10 & -20 & -10 & 0 & 60 \end{bmatrix} \) and \( F = \begin{bmatrix} 1 & 0 & 0 & 5 & 0 & 0 & -29/2 & -255/2 & 0 & -82 \\ 0 & 1 & -2 & 3 & 0 & 6 & 2 & 17 & 0 & 11 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1/2 & -15/2 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & -70 & 0 & -45 \end{bmatrix} \).

It’s a fact that \([C|I_4]\) is row equivalent to \( F \).

1A. Use this information to find a basis for \( \text{col}(C) \).

1B. Find a basis for \( \text{null}(C) \).

1C. Write the last column of \( C \) as a linear combination of the basis vectors in part 1A.
Let \( C = \begin{bmatrix} 10 & 5 & -10 & 65 & 0 & 30 \\ 4 & -7 & 14 & -1 & 0 & -42 \\ -5 & 0 & 0 & -25 & 1 & -2 \\ -8 & 10 & -20 & -10 & 0 & 60 \end{bmatrix} \) and \( F = \begin{bmatrix} 1 & 0 & 0 & 5 & 0 & 0 & -29/2 & -255/2 & 0 & -82 \\ 0 & 1 & -2 & 3 & 0 & 6 & 2 & 17 & 0 & 11 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1/2 & -15/2 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & -70 & 0 & -45 \end{bmatrix} \).

1D. What conditions must a general vector \( \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \) satisfy in order to be in the column space of \( C \)?

1E. Let \( \mathbf{b} = \begin{bmatrix} 10 \\ 4 \\ 0 \\ -8 \end{bmatrix} \). Use the appropriate columns of the row-reduced version of \([C | I_4]\) to express \( \mathbf{b} \) as a linear combination of the basis vectors from 1A.

1F. What is the dimension of the null space of \( C \)?

1G. What is the rank of \( C \)?
2. Define $T : \mathbf{P}_3 \rightarrow \mathbf{P}_2$ by $T(ax^2 + bx + c) = (a + 2b + 3c)x + (a + 3b + 7c)$. You do NOT have to show that $T$ is a linear transformation. Find the kernel of this LT. HINT: what equations do $a, b, c$ need to satisfy? Solve the system using your matrix row reduction techniques. Describe your polynomials in terms of any free variables ($a, b, c$) which arise.

3. Define $T : \mathbf{P}_3 \rightarrow \mathbf{P}_2$ by $T(ax^2 + bx + c) = (abc)x$. Show why $T$ is not a linear transformation, using concrete examples.
4. Consider $S$, our vector space of all sequences $s = \{s_1, s_2, s_3, \ldots\}$ of real numbers.
Let $H = \{s \in S \mid s_1, s_2$ are any real numbers and then $s_3 = s_2 + s_1, s_4 = s_3 + s_2, s_5 = s_4 + s_3, \ldots\}$.

4A. Explicitly, write out the first 6 members of the sequence that begins $(1, 1, \ldots)$.

4B. Is $H$ a subspace of $S$? Prove or disprove.
5. Consider the subspace $H$ of $\mathbb{R}^4$ consisting of all vectors of the form
\[
\begin{bmatrix}
3a + b + 5c \\
2a + 5b - c \\
a + 2b \\
b - c
\end{bmatrix},
\]
where $a, b$ and $c$ are any real numbers. Find a basis for this subspace.

6. Short answers:
6a. What is the dimension of the vector space $\{0\}$?

6b. Suppose $A$ is a 5x5 matrix row-equivalent to $I_5$. What is the dimension of its column space?

6c. What is the dimension of our vector space of sequences $S$?

6d. T/F: If $B = \{v_1, v_2, \ldots, v_p\}$ is a basis for a vector space $V$, then any set of vectors of $V$ with more than $p$ vectors must span $V$.

6e. What’s the dimension of $P_5$?

6f. Suppose $A$ is a 5x5 matrix row-equivalent to $I_5$. What is the dimension of its null space?

6g. T/F: If $B = \{v_1, v_2, \ldots, v_p\}$ is a basis for a vector space $V$, then any set of vectors of $V$ with more than $p$ vectors must be linearly dependent.