1. Let $C = \begin{bmatrix} 10 & 5 & -10 & 65 & 0 & 30 \\ 4 & -7 & 14 & -1 & 0 & -42 \\ -5 & 0 & 0 & -25 & 1 & -2 \\ -8 & 10 & -20 & -10 & 0 & 60 \end{bmatrix}$ and $F = \left[\begin{array}{cccccc|cccc} 1 & 0 & 0 & 5 & 0 & 0 & -29/2 & -255/2 & 0 & -82 \\ 0 & 1 & -2 & 3 & 0 & 6 & 2 & 17 & 0 & 11 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1/2 & -15/2 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & -70 & 0 & -45 \end{array}\right]$.

It's a fact that $[\,C|I_4\,]$ is row equivalent to F.

1A. Use this information to find a basis for col(C).

1B. Find a basis for null(C).

1C. Write the last column of C as a linear combination of the basis vectors in part 1A.

1. CONTINUED for your reference here is the same infor as on the previous page:

$$\text{Let } C = \begin{bmatrix} 10 & 5 & -10 & 65 & 0 & 30 \\ 4 & -7 & 14 & -1 & 0 & -42 \\ -5 & 0 & 0 & -25 & 1 & -2 \\ -8 & 10 & -20 & -10 & 0 & 60 \end{bmatrix} \text{ and } F = \left[\begin{array}{cccccc|cccc} 1 & 0 & 0 & 5 & 0 & 0 & -29/2 & -255/2 & 0 & -82 \\ 0 & 1 & -2 & 3 & 0 & 6 & 2 & 17 & 0 & 11 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1/2 & -15/2 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & -70 & 0 & -45 \end{array}\right].$$

1D. What conditions must a general vector $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ satisfy in order to be in the column space of C?

1E. Let $\mathbf{b} = \begin{bmatrix} 10 \\ 4 \\ 0 \\ -8 \end{bmatrix}$. Use the appropriate columns of the row-reduced version of $[\,C|I_4\,]$ to express $\mathbf{b}$ as a linear combination of the basis vectors from $1A$.

1F. What is the dimension of the null space of C?

1G. What is the rank of C?

2. Define $T : \mathbf{P}_3 \rightarrow \mathbf{P}_2$ by $T(ax^2 + bx + c) = (a + 2b + 3c)x + (a + 3b + 7c)$. You do NOT have to show that T is a linear transformation. Find the kernel of this LT. HINT: what equations do a, b, c need to satisfy? Solve the system using your matrix row reduction techniques. Describe your polynomials in terms of any free variables (a? b? c?) which arise.

3. Define $T : \mathbf{P}_3 \rightarrow \mathbf{P}_2$ by $T(ax^2 + bx + c) = (abc)x$. Show why T is not a linear transformation, using concrete examples.

4. Consider $\mathcal{S}$, our vector space of all sequences $\mathbf{s} = \{s_1, s_2, s_3, \dots\}$ of real numbers.
Let $H = \{\mathbf{s} \in \mathcal{S} \mid s_1, s_2$ are any real numbers and then $s_3 = s_2 + s_1, s_4 = s_3 + s_2, s_5 = s_4 + s_3, \dots\}$.

4A. Explicitly, write out the first 6 members of the sequence that begins $(1, 1, \dots)$.

4B. Is H a subspace of $\mathcal{S}$? Prove or disprove.

5. Consider the subspace H of $\mathbf{R}^4$ consisting of all vectors of the form $\begin{bmatrix} 3a+b+5c \\ 2a+5b-c \\ a+2b \\ b-c \end{bmatrix}$, where a, b and c are any real numbers. Find a basis for this subspace.

6. Short answers:

6a. What is the dimension of the vector space $\{\mathbf{0}\}$?

6b. Suppose A is a 5x5 matrix row-equivalent to I_5. What is the dimension of its column space?

6c. What is the dimension of our vector space of sequences $\mathcal{S}$?

6d. T/F: If $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is a basis for a vector space V, then any set of vectors of V with more than p vectors must span V.

6e. What's the dimension of $\mathbf{P}_5$?

6f. Suppose A is a 5x5 matrix row-equivalent to I_5. What is the dimension of its null space?

6g. T/F: If $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is a basis for a vector space V, then any set of vectors of V with more than p vectors must be linearly dependent.