Math 205 Section B
Test 3 (50 points)

Name: ____

- Check that you have 6 questions on two pages.
- Show all your work to receive full credit for a problem.

1. (9 points) Let $A = \begin{bmatrix} -1 & 3 & -2 \\ 0 & 2 & 2 \end{bmatrix}$. Use this matrix to answer the following questions:

   (a) Find a basis for Col A.

   (b) Col A is a subspace of $\mathbb{R}^2$. Is it possible to find a vector in $\mathbb{R}^2$ that is **not** in Col A? Explain.

2. (7 points) Let V be a vector space of dimension three. The vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, and $\vec{v}_4$ in V are such that $\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\} = V$, and $\vec{v}_1 + 3\vec{v}_2 - 2\vec{v}_3 + \vec{v}_4 = \vec{0}$. Find a basis for V. Explain how the basis you find satisfies the two conditions in the definition of a basis.

3. (8 points) A homogeneous system of seven linear equations in eight unknowns has one free variable. Will the system necessarily have a solution for every possible choice of constants on the right sides of the equations? Briefly explain your answer.

4. (8 points) Define a linear transformation $T : \mathbb{P}_1 \to \mathbb{R}$ by $T(\vec{p}) = \vec{p}(1)$.

   (a) Find a polynomial that spans the kernel of T.

   (b) Is 6 in the range of T? Explain.

5. (8 points) Let $\mathcal{B}$ be the basis of $\mathbb{P}_2$ consisting of the polynomials $1 - t^2$, $t - t^2$, and $2 - 2t + t^2$. Use this basis to answer the following questions:

   (a) Let $\vec{p}(t) = 11t - 3t^2$. Find the coordinate vector of $\vec{p}$ relative to $\mathcal{B}$.

   (b) If $[\vec{q}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, find the polynomial $\vec{q}$. (Your final answer should be a polynomial and not a column vector.)

6. **(10 points)** Let $A = \begin{bmatrix} 1 & 5 & -6 & -7 \\ 2 & 4 & 5 & 2 \\ 0 & 0 & -7 & -4 \\ 0 & 0 & 3 & 1 \end{bmatrix}$. Use this matrix to answer the following questions:

(a) Is $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ an eigenvector of A? Explain.

(b) Is 0 an eigenvalue of A? Explain. If it is an eigenvalue, find a basis and dimension of the corresponding eigenspace.

(c) -1 is an eigenvalue of A. The eigenspace corresponding to the eigenvalue -1 is a subspace of $\mathbb{R}^n$. What is the value of n? (Do not compute the eigenspace.)