1. Find all values of $A$ and $B$ for which $f(x) = e^{Ax} + Bx^2$ is a solution of the differential equation $y'' + 12y = 7y'$.

2. An object moves along a numberline with position $p(t)$, velocity $v(t)$ and acceleration $a(t)$. Suppose that the acceleration is $a(t) = 12t^2 + 16$ (ft/sec$^2$), and the velocity at $t = 1$ is 8 (ft/sec). Suppose also the position at $t = 1$ is 4 ft.

2a. What initial value problem (IVP) is set up to find the formula for $v(t)$?

2b. Solve the IVP in 2a.

2c. What IVP is set up to find the formula for $p(t)$?

2d. Solve the IVP in 2c.

2e. Where is the object at $t = 0$?
3. A cube of ice is melting in such a way that it’s always a perfect cube. The sides are shrinking at 3 inches per hour.
3a. At what rate is the overall surface area decreasing at the moment the sides are 20 inches long? Put correct units on your answer.

3b. At what rate is the distance between the two points A and B on one of the diagonals through the cube's center changing when the side is 20 inches?
4. A rectangle is drawn with its bottom side on the \( x \)-axis and its left side on the \( y \)-axis. The upper right corner of the rectangle is on the graph of \( y = 1/x^2 \). What length should the bottom be in order for the perimeter of the rectangle to be a minimum? Show all your work.
5. Part of the solution of \( \cos(x)y^2 - \sin(3y) = 0 \) is shown here.

5a. From the graph it looks plausible that \((1.4292, 1)\) is a solution; you don't have to check it but do mark this point with the letter \(P\) on your graph. Then carefully draw the tangent line through \(P\), all the way from the left edge to the right edge of the picture.

5b. By using the points where your line meets the left and right edges of the picture, what do you estimate the slope of your line to be? Express your answer as a fraction (rise/run). (The run should be 12 because there are 12 boxes from left to right).

5c. By implicit differentiation, find the formula for \(dy/dx\) at any point along the curve.

5d. To five decimal places, what does your formula in 5c give for the slope of the line tangent in 5a?

5e. Do your answers to 5b and 5d agree?
6. The graph of \( y = \ln x + 3 \sin x \) is shown here.

\[ \text{here, } x = 1.2 \text{ (see part 6b)} \]

6a. There appears to be a root between 3 and 4. Use Newton’s Method to find this root to as many places as your calculator displays, starting with an initial approximation of \( x_0 = 3 \).

Show all your intermediate approximations (only fill out as many as necessary).

\[
\begin{align*}
x_0 &= 3 \\
x_1 &= \\
x_2 &= \\
x_3 &= \\
x_4 &= \\
x_5 &= 
\end{align*}
\]

6b. There’s another root between 0 and 1. If you begin to approximate it starting with an initial approximation of \( x_0 = 1.2 \), you will run into trouble. Use the graph and a geometric understanding of how NM finds the next approximation to explain why.
7A. Find the following limit. Use l’Hopital’s rule if appropriate and use proper phrasing when it comes to writing “∞/∞” in your work!

\[
\lim_{x \to 0} f(x), \quad \text{where } f(x) = \frac{2 \cos(x) - e^x - 1}{x + \sin x}
\]

7B. Find \( f(0.01) \) and \( f(0.001) \). Do these values support your conclusion in 7A?

8. Find the derivative of each of the following. **DO NOT SIMPLIFY.**

8a. \( y = (2^{\sin(\ln(\arctan(x))))}^3 \)

8b. \( y = \cos \left( \frac{\sec(5x) + e^{3x}}{\arcsin(2^x)} \right) \)

8c. \( y = (x^2 + 1)^{\sin x} \)