Math 105: Review for Exam II

1. Find $dy/dx$ for each of the following.
   
   (a) $y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln(2x) + (\ln 2)x + \arctan 2$

   (b) $y = \sqrt{x} \cdot \arctan(5x)$

   (c) $y = \ln(\tan(2^{\cos(x^2)}))$

   (d) $y = \frac{x + e^x}{\cos 4 + \sin^6(6x)}$

   (e) $y = (x^2 + 1)^{\sin x}$
2. Consider the curve defined by $x^3 + y^3 = \frac{9}{2}xy$ (known as the Folium of Descartes).

(a) Find $dy/dx$.

(b) Verify that the point (1,2) is on the curve above.

(c) Find the equation of the tangent line at the point (1,2).

3. Evaluate the following limits.

(a) $\lim_{x \to 1} \frac{x^3 - 1}{x^3 - 1 - 7x}$

(b) $\lim_{x \to 0} \frac{1 - \cos(2x)}{3^x}$

(c) $\lim_{x \to 0} \frac{1 - \cos(4x)}{5x^2}$

(d) $\lim_{x \to \infty} \frac{x^2}{2^x}$
4. Consider the function \( f(x) = x^4 e^x \) with domain all real numbers.

(a) Find the \( x \)-value(s) of all roots (\( x \)-intercepts) of \( f \).

(b) Find the \( x \)- and \( y \)-value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the \( x \)- and \( y \)-value(s) of all global extrema and identify each as a global max or global min.

(d) Find the \( x \)-value(s) of all inflection points.

(e) Sketch \( f \).
5. How would your answers to the previous question change if the domain of $f$ were $[-10, 10]$?

6. **Omit this problem.** Use the Intermediate Value Theorem to explain why $f(x) = x^3 - 4x^2 + 5$ must have a root somewhere on the interval $[1, 2]$.

7. Find an antiderivative of $y = 5^x + x^3 + \cos(2x) + e^3$.

8. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is $288. If the glass for the sides costs $12 per square foot and the opaque material for the bottom costs $3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.