1. In *each* part below, let A be the matrix whose columns are formed by the given set S of vectors.
First, write the RREF form of A next to S.
Second, use RREF(A) to explain whether S spans $\mathbf{R}^3$.
Third, show how RREF(A) indicates if S is linearly independent.
Finally, circle Y or N as the correct answer to the question, is S a basis of $\mathbf{R}^3$.

1A. $S = \left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ -4 \\ 12 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -13 \\ 5 \\ -9 \end{bmatrix} \right\}$ RREF(A): $\begin{bmatrix} 1 & 4 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Span $\mathbf{R}^3$ discussion:

The row of zeros in RREF(A) means that $A\vec{x} = \vec{b}$ will not always be consistent; there are some $\vec{b}$'s in $\mathbb{R}^3$ that cannot be written as linear combinations of the members of S. $\therefore S$ does not span $\mathbb{R}^3$

Linear Independence discussion:

The free variable(s) in RREF(A) tell us that $A\vec{x} = \vec{0}$ has more than just the trivial soln, so S is NOT Linearly Independent

Basis? Y N

1B. $S = \left\{ \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 12 \\ 8 \\ 12 \end{bmatrix}, \begin{bmatrix} 15 \\ 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 16 \\ 7 \\ -13 \end{bmatrix} \right\}$ RREF(A): $\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Span $\mathbf{R}^3$ discussion:

RREF(A) tells us the system $A\vec{x} = \vec{b}$ will be consistent (because no row of 0's) for all $\vec{b} \in \mathbb{R}^3$, that is, every $\vec{b} \in \mathbb{R}^3$ can be written as a L.C. of the members of S, so S DOES span $\mathbb{R}^3$

Linear Independence discussion:

(same as in 1A)

Basis? Y N

quiz continues other side

Quiz 7 side 2
(3/14/08)

1C. $S = \left\{ \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}7\\8\\9\end{bmatrix} \right\}$ RREF(A): $\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}$

Span $\mathbf{R}^3$ discussion:

(Same as in 1A)

Linear Independence discussion:

(Same as in 1A)

Basis? Y N

1D. $S = \left\{ \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix} \right\}$ RREF(A): $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Span $\mathbf{R}^3$ discussion:

(Same as in 1B)

Linear Independence discussion:

here the only solution to $A\vec{x} = \vec{0}$ is the trivial soln since there are no free variables. Thus S is Linearly Independent.

Basis? Y N

---

2. Let $D = \begin{bmatrix} 0 & -1 & -4 & -1 & 1 & 1\\ 2 & 11 & 0 & 4 & -1 & 4\\ -2 & -8 & 12 & 3 & 6 & 5\\ 1 & 5 & -2 & 1 & -1 & 1 \end{bmatrix}$; then D is row equivalent to $\begin{bmatrix} 1 & 0 & -22 & 0 & 12 & 18\\ 0 & 1 & 4 & 0 & -3 & -4\\ 0 & 0 & 0 & 1 & 2 & 3\\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2A. Find a basis for Null(D).

we have these soln's for $A\vec{x}=\vec{0}$:

$$\begin{cases} x_1 = 22x_3 - 12x_5 - 18x_6\\ x_2 = -4x_3 + 3x_5 + 4x_6\\ x_3 = \text{free}\\ x_4 = -2x_5 - 3x_6\\ x_5 = \text{free}\\ x_6 = \text{free} \end{cases}$$

so a basis $\mathcal{B}$ $\left\{ \begin{bmatrix}22\\-4\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-12\\3\\0\\-2\\1\\0\end{bmatrix}, \begin{bmatrix}-18\\4\\0\\-3\\0\\1\end{bmatrix} \right\}$

2B. Express the fifth column of D as a linear combination of the other columns or explain why this is impossible.

... in particular, THIS vector of weights tells us that

$$-12\vec{c}_1 + 3\vec{c}_2 + 0\vec{c}_3 - 2\vec{c}_4 + \vec{c}_5 + 0\vec{c}_6 = \vec{0}$$

(where $\vec{c}_i$ is the ith column of D)

so $\boxed{\vec{c}_5 = 12\vec{c}_1 - 3\vec{c}_2 + 2\vec{c}_4}$