**Math 205B&C** 03/13/09 Quiz 06 page 1 **Name** Suggested Solutions **8am** **1:10 pm**

**1.** Let $B = \begin{bmatrix} 12 & -12 \\ 6 & -5 \end{bmatrix}$.

**1A.** Find the characteristic polynomial of $B$. Show all your steps.

$$\text{char poly}(B) = \det\left(\begin{bmatrix} 12-\lambda & -12 \\ 6 & -5-\lambda \end{bmatrix}\right) = (12-\lambda)(-5-\lambda) + 72$$
$$= -60 + 5\lambda - 12\lambda + \lambda^2 + 72$$
$$= \lambda^2 - 7\lambda + 12$$

**1B.** What, if any, are the eigenvalues of $B$? setting this to 0, we find $0 = (\lambda-4)(\lambda-3)$

$\therefore$ $\lambda = 4$ or $\lambda = 3$ are eig. vals.

**1C.** What number $m$ should the "6" in $B$ be replaced with, so that the resulting matrix has $\lambda = 0$ as an eigenvalue? (The other eigenvalue will be new, too). How did you find $m$?

method 1: choose $m$ so that $\lambda$ is a factor of the char. poly:

$$\begin{vmatrix} 12-\lambda & -12 \\ m & -5-\lambda \end{vmatrix} = -60 - 7\lambda + \lambda^2 + 12m$$

choose $m = 5$; this becomes $-60 - 7\lambda + \lambda^2 + 60$
$$= \lambda^2 - 7\lambda$$
$$= \lambda(\lambda - 7)$$

method 2: $\lambda = 0$ is an eigen value of $B$
$\Leftrightarrow$ $B^{-1}$ doesn't exist
$\Leftrightarrow$ its cols aren't L.I.
$\Leftrightarrow$ one is a multiple of the other [NB b/c there are only 2 columns]
$\Leftrightarrow$ $\begin{bmatrix} 12 & -12 \\ m & -5 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ 5 & -5 \end{bmatrix}$
$\Leftrightarrow$ $m = 5$

**2.** Let $A = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 1 & 2 \\ 1 & -2 & 5 \end{bmatrix}$.

**2A.** It's a fact that $\mathbf{v} = \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix}$ is an eigenvector of $A$. Find the eigenvalue by direct computation of $A\mathbf{v}$.

so $A\vec{v} = \lambda\vec{v}$ for some $\lambda$.

now, $A\vec{v} = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 1 & 2 \\ 1 & -2 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} -10 \\ 10 \\ 10 \end{bmatrix} = 2\begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix} = 2\vec{v}$; ie, $\lambda = 2$

**2B.** It's a fact that $\lambda = 3$ is an eigenvalue of $A$. Find a basis for its eigenspace.

Same as "find a basis for nul$(A - 3I)$

now, $A - 3I = \begin{bmatrix} -1 & 2 & -2 \\ 1 & -2 & 2 \\ 1 & -2 & 2 \end{bmatrix}$; its RREF is $\begin{bmatrix} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and so

a basis is $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ via standard null space techniques.

**2C.** What is the dimension of the eigenspace in (2B)?

2 (= # of elts. in this basis (or any basis of that nullspace))