

NAME _____

I____ II____ III____ IV____ V____ VI____ VII____ VIII____ IX____ X____ TOTAL_____
(10) (10) (10) (10) (10) (10) (10) (10) (10) (10) (10) (100)

March 12
2010

Mathematics 206
Multivariable Calculus
Examination #2

Mr. Haines

(10)I. Suppose $F: \mathbf{R}^2 \rightarrow \mathbf{R}$ with rule $F(x, y) = xy$
and that $G: \mathbf{R} \rightarrow \mathbf{R}^2$ with rule $G(t) = (t, t^2)$. Use the chain rule to:

A. calculate the Jacobian matrix of the function $G \circ F$ at the point $(2, 1)$.

B. calculate the derivative of the function $G \circ F$ at the point $(2, 1)$.

C. calculate the Jacobian matrix of the function $F \circ G$ at 1.

D. calculate the derivative of the function $F \circ G$ at 1.

(10) II. Find the equation of the tangent plane at the point $(0, -1, 2)$ to the surface whose equation is $x^3 + 12y + 3z^2 = 0$.

(10) III. Suppose that $F : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ with rule $F(x, y, z) = (x^3, x^2y^2, xz^2)$.

A. Calculate $\operatorname{div} F$

B. Calculate $\operatorname{curl} F$

(10) IV. Consider the path $f: \mathbb{R} \rightarrow \mathbb{R}^2$ with $f(t) = (e^{-t} \cos t, e^{-t} \sin t)$ for $0 \leq t \leq 1$.

A. Give an integral that computes the total length of this path.

B. Calculate the value of this integral.

(10) V. Let $f(x, y) = x^2 + y^2$.

A. Calculate the First Taylor polynomial for f at $\mathbf{a} = (1, 2)$.

B. Calculate the Second Taylor polynomial for f at $\mathbf{a} = (1, 2)$.

(10) VI. Evaluate: $\int_0^1 \int_0^z \int_0^y z \, dx \, dy \, dz$.

(10) VII. Suppose $f(x, y, z) = x^2y^2 + xy - z - 3y$ and $\mathbf{a} = (1, 1, 1)$.
Calculate the directional derivative of f at \mathbf{a} in the direction parallel to $\mathbf{x} = (1, 2, 2)$

- (10) VIII. Set up but **do not evaluate** an iterated integral that gives the volume of the solid below the surface $x^2 + y^2 + z = 9$ and above the right triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$

- (10) IX. The point $(0, 2)$ is a critical point of $f(x, y) = 2x^2 + x^2y + y^2 - 4y$. Use the Second Derivative Test to determine whether $(0, 2)$ is a local minimum, a local maximum, a saddle point, or none of these.

(10) X. Evaluate the line integral $\int_C \mathbf{F}(x, y, z) \cdot d\mathbf{x}$ where $\mathbf{F}(x, y, z) = (y, z, x)$ if

A) C is the straight line segment from $(0,0,0)$ to $(1,1,1)$.

B) C is the straight line segment from $(1,1,1)$ to $(0,0,0)$.