

NAME _____

I _____ II _____ III _____ IV _____ V _____ VI _____ VII _____ VIII _____ TOTAL _____
(8) (6) (18) (16) (21) (16) (5) (10) (100)

March 12
2009

Mathematics 309a
Abstract Algebra
Examination #2

Mr. Haines

(8) I. Suppose $\phi: \mathbb{Z} \rightarrow \mathbb{Z}_2 \times \mathbb{Z}_4$ is a homomorphism and that $\phi(1) = (1, 2)$

A. $\phi(2) =$ _____ .

B. $\phi(3) =$ _____ .

C. The image of ϕ is _____ .

D. The kernel of ϕ is _____ .

(6) II. Let $H = \langle (123) \rangle$ be the cyclic subgroup of S_7 generated by (123) . Let $\sigma = (36)$ be a transposition in S_7 .

A. $\sigma (123) \sigma^{-1} =$ _____

B. $\sigma H \sigma^{-1} =$ _____

C. $\sigma (1437) \sigma^{-1} =$ _____

(16) IV. If G and H are groups, define $\phi: G \times H \rightarrow H \times G$ by $\phi((g, h)) = (h, g)$.

A. Prove that ϕ is a homomorphism.

B. Prove that ϕ is one-to-one.

C. Prove that ϕ is onto.

D. Why can you now conclude that $G \times H \cong H \times G$?

(21) V. Fill in the blanks:

A. One generator for $Z_8 \times Z_{91}$ is _____ .

B. The order of $(1, 2, 3)(1, 3, 4)$ in S_4 is _____ .

C. The number of left cosets of $\langle 3 \rangle$ in Z_{10} is _____ .

D. The order of the group S_5 is _____ .

E. Express $(142735) \in S_7$ as a product of transpositions _____ .

F. The order of the factor group $(Z_{10} \times Z_6) / \langle (1, 3) \rangle$ is _____ .

G. The subgroup of $(Z_4 \times Z_4) / \langle (0, 2) \rangle$ generated by $(3, 3) + \langle (0, 2) \rangle$ has
_____ elements.

(16) VI. If a is an element of the group G , define $\phi: G \rightarrow G$ by $\phi(g) = aga^{-1}$ for all elements g in G :

A. Prove that if $\phi(g) = e$, then $g = e$.

B. What is the kernel of ϕ ?

C. Prove that $\phi: G \rightarrow G$ is a homomorphism.

D. Prove ϕ is onto by showing that for any element h of G you can construct another element x in G for which $\phi(x) = h$.

(5) VII. Let N be a normal subgroup of S_5 that has 24 elements. It can be shown that S_5/N is a group. Explain why S_5/N is Abelian.

(10) VIII. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones.)

- _____ 1. $\mathbb{Z}_5 \times \mathbb{Z}_2$ is a cyclic group.

- _____ 2. If G and H are finite groups and $\phi: G \rightarrow H$ is a homomorphism, the number of elements in H divides the number of elements in G .

- _____ 3. Every permutation is a cycle.

- _____ 4. The direct product of abelian groups is always abelian.

- _____ 5. S_6 has no cyclic subgroups.

- _____ 6. The composition of two permutations of a set A is always a one-to-one function.

- _____ 7. If G and H are finite groups and $\phi: G \rightarrow H$ is a homomorphism, the number of elements in $\phi[G]$ divides the number of elements in H .

- _____ 8. Every abelian group of order 16 contains a cyclic subgroup of order 8.

- _____ 9. Every cyclic group is Abelian.

- _____ 10. Every finite Abelian group is cyclic.