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March 12,
2009

Mathematics 206a
Multivariable Calculus
Examination #2

Mr. Haines

(10) I. Let C be the curve parametrized by $\vec{f}(t) = \left(t, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}, \frac{t^2}{2} \right)$ starting at $t = 0$ and ending at $t = 2$. Give an equation for the plane that passes through $\vec{f}(1)$ and is perpendicular to the tangent to C at that point.

(10) II. For the vector field $\mathbf{F}(x, y, z) = (xy, -\sin z, 1)$, compute

A. $\text{div}(\mathbf{F}) =$

B. $\text{curl}(\mathbf{F}) =$

(10) III. Find the equation of the plane tangent to the graph of $z = x^2 + y^4 + e^{xy}$ at the point $(1, 0, 2)$.

(10) IV. Derivatives

A. Suppose $\mathbf{f}(x, y, z) = (x + z + y, x^2)$ and $\mathbf{a} = (1, 1, 0)$. Calculate the total derivative of \mathbf{f} at \mathbf{a} .

B. Suppose $\mathbf{g}:\mathfrak{R} \rightarrow \mathfrak{R}^3$ with rule $\mathbf{g}(t) = (6t^2, 3t^3, t)$ and $f:\mathfrak{R}^3 \rightarrow \mathfrak{R}$ with rule xyz . Use the Chain Rule to calculate $(f \circ \mathbf{g})'(1)$.

(10) V. Find the critical points of $f(x, y, z) = x^2 + y^2 + z^2 - x - y - 2z + 1$ and determine whether they are local maxima, local minima, or saddle points.

(10) VI. If $f: \mathfrak{R}^3 \rightarrow \mathfrak{R}$ has rule $f(x, y, z) = x^2 e^{-yz}$, calculate the rate of change of f at $(1, 0, 0)$ in the direction parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

(10) VII. Suppose $f: \mathfrak{R}^3 \rightarrow \mathfrak{R}$ with rule $f(x, y, z) = \sin(x + 2y + 3z)$.

A. Give $p_1(\mathbf{x})$, the first Taylor Polynomial of f at $(0, 0, 0)$.

B. Give $p_2(\mathbf{x})$, the second Taylor Polynomial of f at $(0, 0, 0)$.

(10) VIII. If S is the solid below the surface $z = 4 - x^2 - y^2$ and above the x - y plane, set up but do not evaluate an iterated integral whose value is the triple integral $\iiint_S f(x, y, z) dV$.

(10) IX. Suppose C is the closed parametrized by $f(t) = (3\cos t, 3\sin t)$ starting at $t = 0$ and ending at $t = 2\pi$.

A. If $F : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ with $F(x, y) = x + y$, Compute $\int_c F dL$.

B. If $\mathbf{F} : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ with $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$. Compute $\int_c \vec{F} \bullet d\vec{x}$.

- (10) X. Evaluate the iterated integral and draw the region in the plane determined by the limits on the integral.

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos x} y \sin x \, dy \, dx .$$