NAME:

Instruction: Read each question carefully. Explain ALL your work and give reasons to support your answers. When expressing an infinite series, either write it using summation sign or give at least the first three non-zero terms of the series.

Advice: DON’T spend too much time on a single problem.

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1. For each of the following infinite series, determine whether it converges or diverges. **Justify your answer.**

(10 pts.) (a) 
\[ \sum_{n=1}^{\infty} a_n \]  
where \( a_n = 2 - \left(1 + \frac{1}{n}\right)^2 \).

(10 pts.) (b) 
\[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln n}{n} \]
2. (9 pts.) (a) Determine whether the following series converges or diverges. Justify your answer.
\[
\sum_{n=0}^{\infty} \frac{n!}{2^n (n+1)}
\]

(9 pts.) (b) Find the radius of convergence for the following power series. Justify your answer.
\[
\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n} (x + 2)^n
\]
3. Use the Integral test or the Comparison Test to determine whether each of the following series converges or diverges. **Justify your answer.**

(10 pts.) (a) 

\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^2 + 5}} \]

(10 pts.) (b) 

\[ \sum_{n=0}^{\infty} e^{-n^2} \]
4. (8 pts.) (a) Find the degree 4 Taylor polynomial \( P_4(x; 1) \) for approximating the function \( f(x) = \sqrt{x} \) near \( x = 1 \).

(7 pts.) (b) Find the exact value of the following infinite series, if it converges.

\[
\frac{1}{3.1} - \frac{1}{(3.1)^2} + \frac{1}{(3.1)^3} - \frac{1}{(3.1)^4} + ... 
\]

(6 pts.) (c) Find the exact value of the following infinite series, if it converges.

\[
1 - \frac{2}{1} + \frac{2^2}{2!} - \frac{2^3}{3!} + \frac{2^4}{4!} - \frac{2^5}{5!} + ... 
\]
5. (7 pts.) (a) Find the Taylor series for $\sin 2x$ near $x = 0$.

(7 pts.) (b) Use the result of part (a) to give the Taylor series for $\cos 2x$ near $x = 0$.

(7 pts.) (c) Use the Taylor series for $\frac{\sin 2x}{x}$ to compute $\lim_{x \to 0} \frac{\sin 2x}{x}$. 