1. (18 pts.) Calculate the following:

(a) \( \int (\cos^3 x)(\sin^3 x) \, dx \).

(b) \( \int \frac{dx}{\sqrt{\theta + x^2}} \).
2. (22 pts.) Let \( f(x) = 1/x \).

(a) Calculate the degree-3 Taylor polynomial \( P_3(x) \) for \( f(x) \) at \( x = 2 \).

(b) Find an interval around \( x = 2 \) for which \( P_3(x) \) approximates \( f(x) \) with error at most \( \pm 0.01 \). Explain how you arrived at your answer.

(c) Using Taylor’s Theorem, find the maximum possible error that can occur with this Taylor polynomial on the interval \([1,3]\).
3. (27 pts.) For each of the following, first explain why the integral is improper, and then calculate the integral (or explain why it diverges):

(a) \( \int_{0}^{\infty} \frac{1}{1 + x^2} \, dx. \)

(b) \( \int_{1}^{\infty} \frac{dx}{(x - 3)^2} \, dx. \)

(c) \( \int_{0}^{1} \frac{x}{\sqrt{1 - x^2}} \, dx. \)
4. (18 pts.) Determine if the following integrals are convergent or divergent:

(a) \( \int_{5}^{\infty} \frac{dx}{\sqrt{x} - 1} \)

(b) \( \int_{0}^{\infty} \frac{dx}{x^2 + \sqrt{x}} \). (Hint: Split it up.)
5. (15 pts.) IQ scores (as measured by the Stanford-Binet intelligence test) are normally distributed with a mean of 100 and a standard deviation of 16. (You will need to use the attached table for your calculations.)

(a) Find the probability that someone has an IQ higher than 124.

(b) What is the likelihood of someone having an IQ between 84 and 108?