

MATH106A CALCULUS II - PROF. P. WONG

EXAM II - MARCH 9, 2007

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1.(10 pts.)(a) Find the exact value of the definite integral

$$\int_1^2 x \ln(x^2) dx.$$

(10 pts.)(b) Evaluate the indefinite integral

$$\int \frac{2x}{(x+1)(3x-1)} dx.$$

2.(10 pts.) Find the indefinite integral

$$\int \frac{1}{x^2\sqrt{x^2-1}} dx.$$

(10 pts.)(b) Find the indefinite integral

$$\int \frac{1}{x^2-2x+5} dx.$$

3. Suppose a function f satisfies

$$f(1) = 1, f'(1) = -3, f''(1) = 2, f'''(1) = 3.$$

(10 pts.)(a) Write down the third-degree Taylor polynomial $P_3(x)$ for f based at $x_0 = 1$.

(5pts.)(b) Suppose it is known that for $0 \leq x \leq 2$, $|f^{(4)}(x)| \leq 0.5$. What is the maximum possible error committed by using $P_3(x)$ to estimate $f(x)$ for $0 \leq x \leq 2$?

(5 pts.)(c) Suppose $g(x) = f(x+1)$. Use part (a) to find the third-degree Maclaurin polynomial for g .

4.(10 pts.)(a) Let $f(x) = \cos(2x)$. Find the fourth-degree Maclaurin polynomial $M_4(x)$ for f .

(10 pts.)(b) Solve the following Initial Value Problem

$$y' = (1 + x^2)e^y, \quad y(0) = 0.$$

5. Determine whether each of the following improper integrals converges or diverges. Justify your answers.

(10 pts.)(a)

$$\int_1^{\infty} \frac{1}{(1+x^3)^2} dx$$

[Hint: compare this integral with another improper integral]

(10 pts.)(b)

$$\int_0^1 \frac{1}{2x-1} dx$$