

NAME:

Math 106B - Exam 2 - March 9, 2007

INSTRUCTIONS: Show all of your work and circle your solutions. Cross out any unnecessary work. Calculators are allowed, but notes and books are not.

Reduction formulas for integrals involving trigonometric expressions are given on the last page of this exam.

1. (10 points) Integrate the following. (**Note:** Your final answer should not contain any function compositions of a trigonometric function with an inverse trigonometric function - for example, $\tan(\arcsin x)$ - as you can simplify these.)

$$\int \frac{x^2}{\sqrt{9-x^2}} dx.$$

2. (10 points) Let $f(x) = x \ln x$, and let $P_2(x)$ be the degree-2 Taylor polynomial for $f(x)$ centered at $x_0 = 1$. (**Note:** You do not need to find $P_2(x)$.) According to Taylor's theorem, what is the maximum approximation error committed by $P_2(x)$ on the interval $[0.5, 1.75]$? (**Note:** Explain where your value of K_{n+1} comes from.)

3. (10 points) For a function $f(x)$, suppose $P_6(x)$ is the degree-6 Taylor polynomial for $f(x)$ centered at $x_0 = 3$, where

$$P_6(x) = 4 + 2(x - 3) + \frac{(x - 3)^2}{2} + \frac{2(x - 3)^3}{3} + \frac{(x - 3)^4}{2} + \frac{2(x - 3)^5}{7} + \frac{(x - 3)^6}{2}.$$

- (a) Determine $f^{(5)}(3)$, or explain why you cannot determine it.

- (b) Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 3$.

4. (10 points) Use a comparison to determine whether the following improper integral converges or diverges.

$$\int_1^{\infty} \frac{4}{2\sqrt{x} + 5x^3} dx.$$

5. (10 points) Evaluate the following integral, or explain why it diverges.

$$\int_2^3 \frac{x}{\sqrt{3-x}} dx.$$

6. (10 points) Find all solutions to the differential equation

$$y' = \frac{\sqrt{x} \ln x}{y} + \frac{1}{y}.$$

1	2	3	4	5	6	TOTAL (out of 60)

Reduction formulas for integrals containing trigonometric expressions

$$1. \int \sin^n(x) dx = -\frac{\sin^{n-1}(x) \cos(x)}{n} + \frac{n-1}{n} \int \sin^{n-2}(x) dx, \text{ for } n > 0.$$

$$2. \int \cos^n(x) dx = \frac{\cos^{n-1}(x) \sin(x)}{n} + \frac{n-1}{n} \int \cos^{n-2}(x) dx, \text{ for } n > 0.$$

$$3. \int \tan^n(x) dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x) dx, \text{ for } n \neq 1.$$

$$4. \int \sec^n(x) dx = \frac{\sec^{n-2}(x) \tan(x)}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2}(x) dx, \text{ for } n \neq 1.$$

$$5. \int \sec(x) dx = \ln |\sec(x) + \tan(x)| + C.$$