NAME: ____

Math 205 - Exam 2 - March 8, 2006

Instructions: Show enough work to justify your final answers. When referring to a theorem, give as much information about the theorem as you can.

1. (20 pts.) Consider the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 7 & 6 \\ 2 & 2 & -4 & 10 & 8 \\ 0 & 1 & 0 & 2 & 2 \\ -2 & 2 & 4 & -2 & 6 \end{bmatrix}$, which is row-equivalent to $B = \begin{bmatrix} 1 & 2 & -2 & 7 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) What is the rank of A?

   (b) What is the dimension of the nullspace of A?

   (c) Find a basis for Col A.

   (d) Find a basis for Row A.

   (e) Find a basis for Nul A.

2. (15 pts.) Consider the matrix $M = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ -1 & -1 & 4 \end{bmatrix}$ and vector $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

(a) Calculate $\det M$.

(b) Is $\mathbf{v}$ in Col M?

(c) Is $\mathbf{v}$ in Nul M?

(d) Is $\mathbf{v}$ an eigenvector of M?

(e) Show that $\lambda = 2$ is an eigenvalue of M and calculate its associated eigenspace.

3. (10 pts.) Let $\mathcal{B} = \{1 - t - t^2, -1 + t - t^2, 2 + 2t + 4t^2\}$ be a set of polynomials in $\mathbb{P}_2$.

(a) Show that $\mathcal{B}$ is a basis for $\mathbb{P}_2$.

(b) For $\mathbf{p}(t) = 3 + t + t^2$, find $[\mathbf{p}]_{\mathcal{B}}$.

(c) Is $\mathcal{B}$ a basis for $\mathbb{P}_3$? Explain.

4. (10 pts.) Prove the statement: If A is a $m \times n$ matrix, then Nul A is a subspace of $\mathbb{R}^n$.

5. (15 pts.) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, $B = \begin{bmatrix} a & b & c \\ d+2a & e+2b & f+2c \\ g+3a & h+3b & i+3c \end{bmatrix}$, and $C = \begin{bmatrix} g & h & i \\ 2d & 2e & 2f \\ a & b & c \end{bmatrix}$. Suppose $\det A = 5$.

(a) Find $\det(B)$.

(b) Find $\det(AC)$.

(c) Is AB invertible? Explain.

(d) Are the columns of A linearly independent? Explain.

6. (10 pts.) Suppose A is a $n \times n$ matrix with the property that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$. Prove that each equation $A\mathbf{x} = \mathbf{b}$ has *exactly* one solution.

7. (10 pts.) Let $H = \left\{ \begin{bmatrix} 2a + 3b - c + 5d \\ 2b + c - 6d \\ 2c + d \\ -4c - 2d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$ be a subset of $\mathbb{R}^4$.

(a) Explain why H is a subspace of $\mathbb{R}^4$.

(b) Find a basis for H.

8. (10 pts.) Let A be a 13×15 matrix, and suppose that in trying to solve $A\mathbf{x} = \mathbf{0}$, you find three linearly independent solutions (and there could be more).

(a) Is it possible that $A\mathbf{x} = \mathbf{b}$ always has at least one solution for any vector $\mathbf{b}$ in $\mathbb{R}^{13}$? Why?

(b) Is it possible that $A\mathbf{x} = \mathbf{b}$ ever has exactly one solution? Explain.

1	2	3	4	5	6	7	8	TOTAL