Here are two facts you may find useful:

For problem ONE:

$$\begin{bmatrix} 4 & 2 & 6 & 14 & 24 \\ -1 & 5 & 4 & 10 & 5 \\ 2 & 2 & 4 & 24 & 14 \end{bmatrix} \text{ is row equivalent to } \begin{bmatrix} 1 & 0 & 1 & 0 & 5 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

For problem TWO:

$$\left[\begin{array}{rrr|rrrr} -1 & 5 & 18 & 1 & 0 & 0 & 0 \\ -2 & 5 & 14 & 0 & 1 & 0 & 0 \\ 1 & -3 & -9 & 0 & 0 & 1 & 0 \\ -2 & 3 & 10 & 0 & 0 & 0 & 1 \end{array}\right] \text{ is row equivalent to } \left[\begin{array}{rrr|rrrr} 1 & 0 & 0 & 0 & -3/4 & -2 & -3/4 \\ 0 & 1 & 0 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & 0 & -3/4 & -1 & 1/4 \\ 0 & 0 & 0 & 1 & 11/4 & 6 & -1/4 \end{array}\right]$$

1. Suppose $T : \mathbf{R}^3 \to \mathbf{R}^4$ is a linear transformation and it's given that

$$T\left(\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 8 \\ 3 \\ 8 \\ -11 \end{bmatrix}, \quad T\left(\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -14 \\ 29 \\ 38 \\ 27 \end{bmatrix}, \quad \text{and} \quad T\left(\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} -6 \\ 32 \\ 46 \\ 16 \end{bmatrix}.$$

1A. Express $\mathbf{x}_1 = \begin{bmatrix} 14 \\ 10 \\ 24 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$, or explain why you can't.

1B. Find $T(\mathbf{x}_1)$ or explain why you can't, based on what you know about T.

1C. Express $\mathbf{x}_2 = \begin{bmatrix} 24 \\ 5 \\ 14 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$, or explain why you can't.

1D. Find $T(\mathbf{x}_2)$ or explain why you can't, based on what you know about T.

2. Suppose $T : \mathbf{R}^3 \to \mathbf{R}^4$ is a linear transformation with standard matrix $A = \begin{bmatrix} -1 & 5 & 18 \\ -2 & 5 & 14 \\ 1 & -3 & -9 \\ -2 & 3 & 10 \end{bmatrix}$.

2A. Find $T\left(\begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}\right)$

2B. Is T *onto* $\mathbf{R}^4$? Explain your answer.

2C. Are there any conditions that a vector $\mathbf{b}$ must satisfy in order to be in the image of T? If so, what are they?

2D. Give an example of a vector which is *not* in the image of T, or explain why this cannot be done.

2E. Is T one-to-one? Explain your answer.

2F. Give three different vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, each satisfying $T(\mathbf{x}) = \mathbf{0}$, or explain why this cannot be done.

3. Suppose an economy is modeled with four sectors A, B, C, and D. Suppose that the output of D is evenly divided (consumed) by all four sectors, while A and B each buy 75% of each other's output yet keep none of their own (respectively). Suppose the remainder of A's output is purchased by C. Suppose that none of C's output is sold to B and *vice versa*, while A and D each use 40% of C's ouput.

3A. Find the exchange table for this economy. You may assume all columns sum to one.

3B. Find the complete set $\{P_A, P_B, P_C, P_D\}$ of equilibrium solutions for this economy. Write down any system of equations and augmented matrices you use in solving this problem.

3C. Suppose P_D is 100 dollars. Rank all four equilibrium prices from least to greatest.

4. Let $M = \begin{bmatrix} 4 & a & 0 \\ 0 & 3 & b \\ 0 & 0 & 1 \end{bmatrix}$.

4A. In terms of a and b, find the inverse of M using the "$[A|I] \sim [I|A^{-1}]$" algorithm discussed in class and show all your steps.

4B. Find the determinants of each of the following matrices and write your answers in the boxes.

3M ☐

M^3 ☐

$\begin{bmatrix} 4 & a & 0 \\ 4 & 3+a & b \\ 8 & 8a & 1 \end{bmatrix}$ ☐

$\begin{bmatrix} 4 & a & -1 \\ 0 & 3 & b \\ 8 & 2a+3 & b-2 \end{bmatrix}$ ☐

$2M + 3I_3$ ☐

$M^{-1}M^T$ ☐

5. Consider the vector space $\mathbf{F}$ of functions $f : \mathbf{R} \rightarrow \mathbf{R}$ which are continuous everywhere, as discussed in class. Let H consist of all members of $\mathbf{F}$ which are differentiable at $x = 1$ and furthermore, the line tangent to the graph of any member of H is horizontal at $x = 1$.

5A. What does this last condition say that $h'(1)$ equals, for any member $\mathbf{u} = h(x)$ which belongs to H?

5B. Which, if any, of the following functions belong to H? Explain!

a) e^{x-1}

b) $p(x) = 2x^2 - 4x + 8$

5C. PROVE that H is a subspace of $\mathbf{F}$.

6. Let $\mathbf{P}_3$ be the vector space of polynomials of degree three-or-less, and let H be all members of $\mathbf{P}_3$ which have a slope of 1 when $x = 1$.

6A. Find three members of H, one of degree one, one of degree two, and one of degree three.
CIRCLE your answers

a)

b)

c)

6B. Show that H is not closed under vector addition.