Math 205B&C 03/06/09 Quiz 05 page 1 **Name** Suggested soln's **8am** **1:10 pm**

1. Suppose $A^\triangle = \begin{bmatrix} 12 & 3 & 9 & -18 \\ 0 & 3 & 4 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, and A is some 4×4 matrix.

Suppose that $A \sim A_1 \sim A_2 \sim A_3 \sim A_4 = A^\triangle$, and the following row operations are applied sequentially to turn A into $A^\triangle$:

Matrix A_1: Row 3 of A is divided by 4.

Matrix A_2: In A_1, 2 copies of Row 4 are added to Row 2.

Matrix A_3: In A_2, Row 4 is multiplied by 3.

Matrix $A_4 = A^\triangle$: Rows 1 and 4 of A_3 are swapped.

1A. Find $\det(A^\triangle)$. Since $A^\triangle$ is upper triangular, $\det(A^\triangle)$ = the product of its main diagonal elts $= 12\cdot3\cdot1\cdot4 = \boxed{144}$

1B. Find $\det(A)$. The cumulative effects of the row operations tell us

$\det(A^\triangle) = (-1)\cdot(3)\cdot(1)\cdot(\tfrac{1}{4})\cdot\det(A)$ so $\det(A) = 144\cdot(-1)(\tfrac{1}{3})(1)(4) = \boxed{-192}$

1C. Find and label the original matrix A, and matrices A_1, A_2 and A_3.

We need to apply the inverses of the above 4 row operations to $A^\triangle$, in reverse order. Changes in **bold**:

so, if $A^\triangle = A_4 = \begin{bmatrix} 12 & 3 & 9 & -18 \\ 0 & 3 & 4 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, then $A_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{4} \\ 0 & 3 & 4 & 7 \\ 0 & 0 & 1 & 6 \\ \mathbf{12} & \mathbf{3} & \mathbf{9} & \mathbf{-18} \end{bmatrix}$, so $A_2 = \begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 3 & 4 & 7 \\ 0 & 0 & 1 & 6 \\ \mathbf{4} & \mathbf{1} & \mathbf{3} & \mathbf{-6} \end{bmatrix}$; $A_1 = \begin{bmatrix} 0 & 0 & 0 & 4 \\ \mathbf{-8} & \mathbf{1} & \mathbf{-2} & \mathbf{19} \\ 0 & 0 & 1 & 6 \\ 4 & 1 & 3 & -6 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 & 0 & 4 \\ -8 & 1 & -2 & 19 \\ \mathbf{0} & \mathbf{0} & \mathbf{4} & \mathbf{24} \\ 4 & 1 & 3 & -6 \end{bmatrix}$

1D. Suppose those same four operations were applied to $A^\triangle$ itself (so the first operation is "Row 3 of $A^\triangle$ is divided by 4", and so on). Find the determinant of the resulting matrix M (you do *not* need to find M).

Now the cumulative effects of the row ops are applied to $\det(A^\triangle)$, so that $\det(M) = -1\cdot3\cdot1\cdot\tfrac{1}{4}\det(A^\triangle) = -\tfrac{3}{4}\cdot144 = -108$

2. Suppose $B = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$, and $\det(B) = 3$. Find each of the following:

2A. $\det(B \cdot B) = (\det B)(\det B) = 3\cdot3 = 9$

2B. $\det(B^5)$

$= \det(B\cdot B\cdot B\cdot B\cdot B)$

$= \det(B)\cdot \cdots \cdot\det(B) = 3\cdot\cdots\cdot3 = 3^5 = 243$

2C. $\begin{vmatrix} 5p & 5q & 5r \\ a & b & c \\ x-7a & y-7b & z-7c \end{vmatrix}$ 3 row operations applied here in some order are a row swap, an addition of a multiple of one row to another, and a multiplication of one row by 5. Cumulative effects give that this det is $(-1)(1)(5)\det(B) = -15$

2D. $\det(5B)$ ← each of B's 3 rows gets multiplied by 5 $\therefore \det(5B) = 5\cdot5\cdot5\cdot3 = 375$

2E. $\det(B+B) \neq \det(B)+\det(B)$, that's for sure. But $\det(B+B) = \det(2B)$

$= 2\cdot2\cdot2\cdot3 = 24$ (see 2D!)

2F. $\det(B^T)$ $\to = 3$

2G. $\det(B^{-1})$ $\to \tfrac{1}{3}$

2H. Is B singular or nonsingular? Explain!

Non singular. $\det(B) \neq 0 \Rightarrow B^{-1}$ exists, which is what non singular means.

2I. Do the columns of B form a linearly independent set? Explain!

Since B^{-1} exists, the only soln to $B\vec{x} = \vec{0}$ is $\vec{x} = \vec{0}$, so the only L.C. of these 3 columns which produces 0 is $0\vec{c}_1 + 0\vec{c}_2 + 0\vec{c}_3$. $\therefore$ L.I.