

1. Suppose $A^\Delta = \begin{bmatrix} 12 & 3 & 9 & -18 \\ 0 & 3 & 4 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, and A is some 4×4 matrix.

Suppose that $A \sim A_1 \sim A_2 \sim A_3 \sim A_4 = A^\Delta$, and the following row operations are applied sequentially to turn A into A^Δ :

Matrix A_1 : Row 3 of A is divided by 4.

Matrix A_2 : In A_1 , 2 copies of Row 4 are added to Row 2.

Matrix A_3 : In A_2 , Row 4 is multiplied by 3.

Matrix $A_4 = A^\Delta$: Rows 1 and 4 of A_3 are swapped.

1A. Find $\det(A^\Delta)$.

1B. Find $\det(A)$.

1C. Find and label the original matrix A , and matrices A_1 , A_2 and A_3 .

1D. Suppose those same four operations were applied to A^Δ itself (so the first operation is “Row 3 of A^Δ is divided by 4”, and so on). Find the determinant of the resulting matrix M (you do *not* need to find M).

2. Suppose $B = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$, and $\det(B) = 3$. Find each of the following:

2A. $\det(B \cdot B)$

2B. $\det(B^5)$

2C. $\begin{vmatrix} 5p & 5q & 5r \\ a & b & c \\ x - 7a & y - 7b & z - 7c \end{vmatrix}$

2D. $\det(5B)$

2E. $\det(B + B)$

2F. $\det(B^T)$

2G. $\det(B^{-1})$

2H. Is B singular or nonsingular? Explain!

2I. Do the columns of B form a linearly independent set? Explain!