NAME______________________________________

I_____II_____III_____IV_____V_____VI_____VII_____VIII_____IX_____TOTAL ________

March 6 2008

Mathematics 309a
Abstract Algebra
Examination #2

Mr. Haines

(10) I. Define any <u>two</u> of these terms. Use a complete, mathematically correct sentence for each definition.

cycle
the alternating group on n letters
left coset of a subgroup of a group $\langle G, *\rangle$

A.

B.

(4) II. Complete the following statement of **Lagrange's Theorem**:

If G is a finite group and H is a subgroup of G, then ...

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(16) III. Give examples of:

A. A homomorphism from $\mathbb{Z}_3$ to $\mathbb{Z}_6$ whose kernel is $\{0\}$.

B. A non-trivial homomorphism from $\mathbb{Z}_8$ to $\mathbb{Z}_6$

C. A non-abelian group with 12 elements.

D. A cyclic group that is isomorphic to the direct product of two of its non-trivial subgroups.

(21) IV. Fill in the blanks:

A. One generator for $\mathbb{Z}_8 \times \mathbb{Z}_9$ is ________________________ .

B. The order of (1, 2, 3)(2, 3, 4) in S_4 is _________________________ .

D. The number of left cosets of $\langle 5 \rangle$ in $\mathbb{Z}_{10}$ is ____________________.

E. The order of the group S_6 is ______________________ .

G. Express $(1, 4, 2, 5, 3, 7) \in S_7$ as a product of transpositions __________________ .

H. The order of the factor group $(\mathbb{Z}_{11}\times\mathbb{Z}_6)/<(1,3)>$ is _____________________.

I. The subgroup of $(\mathbb{Z}_4 \times \mathbb{Z}_8)/<(0,2)>$ generated by $(3, 3) + <(0,2)>$ has ________________________ elements.

(9) V. If $\langle G, *\rangle$ is an abelian group with identity e, define $H = \{x \in G \mid x * x = e\}$.
If $G = \mathbb{Z}_2 \times \mathbb{Z}_4$, what is H ?

(10) VI. Suppose $\phi : G \rightarrow G'$ is a homomorphism and that $\ker(\phi) = \{e\}$. Let $x, y \in G$ and suppose $\phi(x) = \phi(y)$. Prove that $x = y$. You may use all the other results we have proven for homomorphisms, such as $\phi(e) = e'$, $(\phi(y))^{-1} = \phi(y^{-1})$, etc. [Hint: Consider $\phi(xy^{-1})$.]

(10) VII. If H is a subgroup of G, a an element of G, and $a\mathrm{H} = \mathrm{H}a$, prove that if h is an arbitrary element of H, then aha^{-1} is an element of H .

(10) VIII. Draw a regular pentagon and label its vertices with the integers 1, 2, 3, 4, and 5 . The symmetries of the pentagon form a group of order 10, sometimes denoted D_5 and called the **5th dihedral group**. Use cycle notation to express each of these symmetries as a permutation of the vertices of the pentagon.

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(10) IX. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones.)

_____ 1. $\mathbb{Z}_5 \times \mathbb{Z}_{25}$ is a cyclic group.

_____ 2. The number of elements in any subgroup of a finite group G divides the number of elements in G .

_____ 3. Every permutation is a cycle.

_____ 4. The direct product of abelian groups is always abelian.

_____ 5. S_6 has no cyclic subgroups.

_____ 6. The composition of two permutations of a set A is always a permutation of A .

_____ 7. Every left coset of a subgroup of a group G is also a subgroup of G .

_____ 8. Every abelian group of order 8 contains a cyclic subgroup of order 8 .

_____ 9. Every cycle is a permutation

_____10. Every finite group of prime order is cyclic