

NAME \_\_\_\_\_

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March 6,  
2008

Mathematics 206a  
Multivariable Calculus  
Examination #2

Mr. Haines

(10)I. Derivatives

A. Suppose  $\mathbf{f}(x, y, z) = (x + e^z + y, yx^2)$  and  $\mathbf{a} = (1, 1, 0)$ . Calculate the total derivative of  $\mathbf{f}$  at  $\mathbf{a}$ .

B. Suppose  $\mathbf{g}:\mathfrak{R} \rightarrow \mathfrak{R}^3$  with rule  $\mathbf{g}(t) = (6t, 3t^2, t^3)$  and  $f:\mathfrak{R}^3 \rightarrow \mathfrak{R}$  with rule  $f(x, y, z) = e^{xyz}$ . Use the Chain Rule to calculate  $(f \circ \mathbf{g})'(1)$ .

(10)II. If  $f: \mathfrak{R}^2 \rightarrow \mathfrak{R}$  has rule  $f(x, y) = \ln \sqrt{x^2 + y^2}$ , calculate the directional derivative of  $f$  at  $(2, 0)$  in the direction parallel to the vector  $2\mathbf{i} + \mathbf{j}$ .

(10) III. For the vector field  $\mathbf{F}(x, y, z) = (x^2 y, z, xyz)$

A.  $\text{div}(\mathbf{F}) =$

B.  $\text{curl}(\mathbf{F}) =$

(10) IV. Find the equation of the tangent plane to the surface  $x^2 + 2y^2 + 3xz = 10$  at the point  $(1, 2, 1/3)$ .

(10) V. Suppose that a mountain has the shape of an elliptic paraboloid with equation  $z = 4 - x^2 - 2y^2$ . If a marble were released at  $(1, 1, 1)$ , in what direction in the  $xy$ -plane would it begin to roll?

(10) VI. Find the critical points of  $f(x, y) = e^{1+x^2-y^2}$  and determine whether they are local maxima, local minima, or saddle points.

(10) VII. Let  $C$  be the path in  $\mathfrak{R}^4$  parametrized by  $f(t) = (\cos t, \sin t, \cos 2t, \sin 2t)$  starting at  $t = 0$  and ending at  $t = \pi$ . Compute the length of  $C$ .

(10) VIII. Suppose  $C$  is the helical path parametrized by  $f(t) = (\cos t, \sin t, t)$  starting at  $t = 0$  and ending at  $t = 2\pi$ .

A. If  $F : \mathfrak{R}^3 \rightarrow \mathfrak{R}$  with  $F(x, y, z) = x + y + z^2$ , Compute  $\int_C F dL$ .

B. If  $\mathbf{F} : \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$  with  $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ . Compute  $\int_C \vec{F} \cdot d\vec{x}$ .

(10) IX. If  $S$  is the solid bounded below by the paraboloid  $z = x^2 + y^2$  and above by the plane  $z = 1$ , set up but do not evaluate an iterated integral whose value is the triple integral

$$\iiint_S f(x, y, z) dV .$$

(10) X. Compute the value of the triple integral of  $f(x, y, z) = xyz$  over the box  $[0,1] \times [0,1] \times [0,1]$ .