(15) I. Suppose \( R \) is the region in the xy-plane bounded by \( x = 0, \ x = 2, \ y = 0, \) and \( y = x^2 \). Suppose \( f(x) = x^2 + y^2 \).

Set up and evaluate two iterated integrals which give the volume of the solid above the region \( R \) and under the surface \( z = f(x, y) \):

A. Integrate first with respect to \( y \) and then with respect to \( x \).

B. Integrate first with respect to \( x \) and then with respect to \( y \).
(10) II. Suppose \( f(x, y, z) = (x^2 y^3, xyz) \) and \( a = (1, 1, 1) \)

A) Calculate the Jacobian matrix of \( f \) at \( a \).

B) Calculate the total derivative of \( f \) at \( a \).
(10) III.  Suppose $F : \mathbb{R}^2 \to \mathbb{R}^3$ with rule $F(x, y) = (xy, y, x)$ and $G : \mathbb{R}^3 \to \mathbb{R}^1$ with rule $G(x, y, z) = x^3 + y^3$.

A. Calculate the Jacobian matrix of the function $F$ at the point $(1, 1)$.

B. Calculate $F(1, 1)$.

C. Calculate the Jacobian matrix of the function $G$ at the point $F(1, 1)$.

D. Calculate the Jacobian matrix of the function $G \circ F$ at the point $(1, 1)$. 
(10) IV. Suppose \( f : \mathbb{R}^3 \rightarrow \mathbb{R} \) with rule \( f(x,y,z) = x + y + z^5 \)

A) Calculate the gradient of \( f \) at (1, 1, 1).

B) Calculate the directional derivative of \( f \) at (1, 1, 1) in the direction parallel to the vector (1, 2, 3).

(10) V. For the vector field \( \mathbf{F}(x,y,z) = (yz, xz, xy) \)

A) \( \text{div} (\mathbf{F}) = \)

B) \( \text{curl} (\mathbf{F}) = \)
(15) VI. For the function \( f(x, y, z) = x^3 + y^2 + z \) at the point (0, 0, 0) compute:

A) The Hessian matrix.

B) The Hessian form.

C) The second-degree Taylor polynomial.
(10) VII. Compute the critical points, if any, or the function $f : \mathbb{R}^2 \to \mathbb{R}$ with rule

$$f(x, y) = (x^2 - 1)e^y.$$ If there are any, test them for local extrema. If there are none, state why.
(10) VIII. C is a curve in Euclidean 3-space connecting the points (0, 0, 0) and (1, 1, 1) parametrized by \( f : [0,1] \rightarrow \mathbb{R}^3 \) with rule \( f(t) = (t^2, t^2, t^2) \).

A. Compute \( f(0) \).

B. Compute \( f(1) \).

C. Compute the length of the curve C parametrized by \( f \).
(10) IX. C is the straight line segment in Euclidean 3-space that connects the points \((1, 2, 0)\) and \((2, 3, 2)\).

\[ \mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3 \text{ has rule } \mathbf{F}(x, y, z) = (-x + y + z, x - y + z, x + y - z). \]

Compute \( \int_C \mathbf{F} \cdot d\mathbf{x} \).