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I ___ II ___ III ___ IV ___ V ___ VI ___ VII ___ VIII ___ IX ___ TOTAL _____

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Mathematics 206a
Multivariable Calculus
Examination #2

Mr. Haines

(15) I. Suppose R is the region in the xy -plane bounded by $x = 0$, $x = 2$, $y = 0$, and $y = x^2$.
Suppose $f(x, y) = x^2 + y^2$.

Set up and evaluate two iterated integrals which give the volume of the solid above the region R and under the surface $z = f(x, y)$:

A. Integrate first with respect to y and then with respect to x .

B. Integrate first with respect to x and then with respect to y .

(10) II. Suppose $\mathbf{f}(x, y, z) = (x^2 y^5, xyz)$ and $\mathbf{a} = (1, 1, 1)$

A) Calculate the Jacobian matrix of \mathbf{f} at \mathbf{a} .

B) Calculate the total derivative of \mathbf{f} at \mathbf{a}

(10) III. Suppose $\mathbf{F} : \mathfrak{R}^2 \rightarrow \mathfrak{R}^3$ with rule $\mathbf{F}(x, y) = (xy, y, x)$
and $\mathbf{G} : \mathfrak{R}^3 \rightarrow \mathfrak{R}^1$ with rule $\mathbf{G}(x, y, z) = x^3 + y^3$.

A. Calculate the Jacobian matrix of the function \mathbf{F} at the point $(1, 1)$.

B. Calculate $\mathbf{F}(1, 1)$.

C. Calculate the Jacobian matrix of the function \mathbf{G} at the point $\mathbf{F}(1, 1)$.

D. Calculate the Jacobian matrix of the function $\mathbf{G} \circ \mathbf{F}$ at the point $(1, 1)$.

(10) IV. Suppose $f: \mathfrak{R}^3 \rightarrow \mathfrak{R}$ with rule $f(x, y, z) = x + y + z^5$

A) Calculate the gradient of f at $(1, 1, 1)$.

B) Calculate the directional derivative of f at $(1, 1, 1)$ in the direction parallel to the vector $(1, 2, 3)$.

(10) V. For the vector field $\mathbf{F}(x, y, z) = (yz, xz, xy)$

A) $\text{div}(\mathbf{F}) =$

B) $\text{curl}(\mathbf{F}) =$

(15) VI. For the function $f(x, y, z) = x^3 + y^2 + z$ at the point $(0, 0, 0)$ compute:

A) The Hessian matrix.

B) The Hessian form.

C) The second-degree Taylor polynomial.

- (10) VII. Compute the critical points, if any, or the function $f : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ with rule $f(x, y) = (x^2 - 1)e^y$. If there are any, test them for local extrema. If there are none, state why.

(10) VIII. C is a curve in Euclidean 3-space connecting the points $(0, 0, 0)$ and $(1, 1, 1)$ parametrized by $\mathbf{f} : [0,1] \rightarrow \mathfrak{R}^3$ with rule $\mathbf{f}(t) = (t^2, t^2, t^2)$.

A. Compute $\mathbf{f}(0)$.

B. Compute $\mathbf{f}(1)$.

C. Compute the length of the curve C parametrized by \mathbf{f} .

(10) IX. C is the straight line segment in Euclidean 3-space that connects the points $(1, 2, 0)$ and $(2, 3, 2)$

$\mathbf{F} : \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$ has rule $\mathbf{F}(x, y, z) = (-x + y + z, x - y + z, x + y - z)$.

Compute $\int_C \mathbf{F} \bullet d\mathbf{x}$.