1. Let A be the matrix below. Recall that the *null space* of A is the set of solutions of the equation $A\mathbf{x} = \mathbf{0}$. Recall also that the *column space* of A consists of the vectors $\mathbf{b}$ for which the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

$$A = \begin{bmatrix} 1 & 0 & 6 & 4 & 0 \\ 0 & 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 4 & -3 & 33 & 10 & 0 \end{bmatrix}$$

1A. Which of the following column vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ is in the column space of A? Hint: I'd row reduce the matrix $[A|\mathbf{b}_1|\mathbf{b}_2]$ all at once, rather than use two separate augmented matrices. *Explain your answers!*

$$\mathbf{b}_1 = \begin{bmatrix} 4 \\ -2 \\ 5 \\ 22 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 4 \\ -2 \\ 5 \\ 18 \end{bmatrix}$$

1B. Find all the vectors in the null space of A, writing them in terms of a linear combination of appropriate column vectors using free variables.

2. Let A be the following matrix. Find an LU factorization of A by the method we have used in class. Do not multiply rows by scalars or swap rows in finding U; only add/subtract multiples of one row to/from another.

$$A = \begin{bmatrix} 3 & -5 & 7 \\ -3 & 7 & -11 \\ 9 & -9 & 9 \end{bmatrix}$$

3. Suppose A is a matrix whose LU factorization is

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Use the LU factorization of A to solve the equation $A\mathbf{x} = \begin{bmatrix} 11 \\ 35 \\ -17 \end{bmatrix}$. Credit on this problem is given for showing the proper steps and corresponding intermediate results.

Now, find the matrix A. Verify that your solution works.

4. Use the method discussed in class to find the inverse of the following matrix. Credit on this problem is given for showing the proper steps and corresponding intermediate results.

$$A = \begin{bmatrix} -2 & 1 & -6 \\ 4 & 0 & 12 \\ 12 & 0 & 39 \end{bmatrix}$$

Hint! I'd start by swapping the first and second rows, then dividing the new first row by...

5. Find both the determinant *and* the inverse of this matrix A:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

6. Find the determinant of the following matrix B, by judicious choices of which rows and/or columns to use at each stage. *Watch your signs!*

$$B = \begin{bmatrix} 0 & 0 & 5 & 4 \\ 2 & 9 & -6 & 3 \\ 0 & -3 & 39 & 8 \\ 0 & 0 & 12 & 8 \end{bmatrix}$$

7. Let A and B be as in problems 5 and 6, respectively. Find each of the following:

7a. Det(AB)

7b. Det(B^T)

7c. Det$((BA)^{-1})$

8. Is the subset H of $\mathbf{R}^2$ consisting of all column vectors of the form $\begin{bmatrix} a \\ a^2 \end{bmatrix}$ a sub*space* of $\mathbf{R}^2$? Prove it is a subspace or explain why it isn't.

9. Let S be the vector space of all sequences $\mathbf{s} = (s_1, s_2, s_3, \ldots)$ of real numbers as discussed in class. Let H be the subset of S consisting of sequences where all but a finite number of terms in the sequence are 0. For example: $\mathbf{u} = (2, 5, 0, 0, 7, 0, 0, 0, 0, 0, 0, 0, \ldots)$ is in H because only a finite number (three) of its terms aren't zero.

9a. Is $\mathbf{s} = (1, 2, 4, 8, 16, 32, 64 \ldots)$ in H? Explain!

9b. Is $\mathbf{s} = (1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0 \ldots)$ in H? Explain!

9c. Is the zero-vector of S in H? Explain!

9d. Is $\mathbf{v} = (0, 2, 4, 6, 8, 10, 0, 0, 0, 0, 0, 0, 0, \ldots)$ in H? Explain!

9e. Find $\mathbf{u} + \mathbf{v}$. Is $\mathbf{u} + \mathbf{v}$ in H? *Explain!*

9f. Give an good argument that in general, the sum of any two vectors in H is again in H, or else find a counterexample.

9g. If $\mathbf{t}$ is in H and α is a scalar, does it follow that $\alpha\mathbf{t}$ is in H also? Explain.

9h. Is H a subspace of S? If so, which parts (a,b,c,etc) above show it. If not, explain.