LINEAR ALGEBRA EXAM 2 March 3, 2004

Name: ______________________________

While the final answer is important, you earn points for all the work leading to that answer, as well as the answer itself. Show all your steps clearly so you will be eligible for the most partial credit. Good luck!

1.) (15 pts.) Find an LU factorization of the matrix

$$\begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}.$$

2.) (10 pts.) If G is an $n \times n$ matrix and the equation $G\mathbf{x} = \mathbf{y}$ has more than one solution for some $\mathbf{y}$ in $\mathbb{R}^n$, can the columns of G span $\mathbb{R}^n$? Why or why not?

3.) (10 pts.) Show that if A is invertible, then $\det A^{-1} = \dfrac{1}{\det A}$.

4.) (15 pts.) Find the determinant below by combining the methods of row reduction and cofactor expansion. YOU NEED TO SHOW ALL YOUR STEPS - a calculator result earns no credit.

$$\begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix}$$

5.) (10 pts.) **True or False:** A vector space is also a subspace. If true, explain why. If false, correct the statement to make it true.

6.) (10 pts.) Assume the matrices below are partitioned conformably for block multiplication. Compute the product

$$\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix} \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}.$$

7.) (15 pts.) Determine whether $\mathbf{w}$ is in the column space of A, the null space of A, or both, where

$$\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -3 \end{bmatrix}, \qquad A = \begin{bmatrix} 7 & 6 & -4 & 1 \\ -5 & -1 & 0 & -2 \\ 9 & -11 & 7 & -3 \\ 19 & -9 & 7 & 1 \end{bmatrix}.$$

Note: I encourage you to use your calculator on this question. However, use words to describe what you do with the calculator, and to explain how your steps on the calculator show whether $\mathbf{w}$ is in the column space or null space of A.

8.) (15 pts.) Let W be the set of all vectors of the form $\begin{bmatrix} c-2a \\ a+b+c \\ 0 \\ 4a+3b \end{bmatrix}$, where a, b, and c represent arbitrary real numbers. Either find a set S of vectors that spans W or give an example to show that W is *not* a vector space.