Math 205A Quiz 06 page 1 February 29, 2008 NAME suggested solutions

1. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and suppose $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5.$

Find each of the following and write your final answer in the box provided.

1a. $\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$ $\boxed{5}$

it takes two sets of row swaps to produce this matrix from A

1b. $\begin{vmatrix} a & b & c \\ d+2a & e+2b & f+2c \\ g+4d+7a & h+4e+7b & i+4f+7c \end{vmatrix}$ $\boxed{5}$

(all row operations involve adding multiples of other rows to certain rows)

1c. $\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g/4 & h/4 & i/4 \end{vmatrix}$ $\boxed{15/4}$

1d. $\det(3A)$ $\boxed{135}$

since each of the three rows of A is multiplied by a factor of 3, the det of 3A is $3 \cdot 3 \cdot 3 \det(A) = 27 \cdot 5 = 135$

1e. $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$ $\boxed{5}$

this matrix is A^T, so has same det

1f. $\begin{vmatrix} g & d & a \\ h & e & b \\ i & f & c \end{vmatrix}$ $\boxed{-5}$

see reason below in #3

1g. $\begin{vmatrix} d & e & f \\ a+g & b+h & c+i \\ d & e & f \end{vmatrix}$ $\boxed{0}$

by subtracting r_3 from r_1, this matrix has same det as $\begin{bmatrix} 0 & 0 & 0 \\ a+g & b+h & c+i \\ d & e & f \end{bmatrix}$, which has det 0

1h. $\det(A^{-1})$ $\boxed{1/5}$ $= \dfrac{1}{\det(A)}$

2. By hand, find the determinant of the following matrix, using "cofactors" and taking advantage of zeros wherever possible. *Show all your steps.*

$$\begin{bmatrix} 4 & -9 & 8 & 1 \\ 0 & 0 & 5 & 0 \\ 6 & 3 & 10 & 2 \\ 7 & 0 & 2 & 3 \end{bmatrix}$$

there are lots of possible paths to completion of this problem. Here's one:

$$\det(\text{THIS}) = -5 \begin{vmatrix} 4 & -9 & 1 \\ 6 & 3 & 2 \\ 7 & 0 & 3 \end{vmatrix} = -5\left(7\begin{vmatrix} -9 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 4 & -9 \\ 6 & 3 \end{vmatrix}\right)$$

$$= -5\left(7 \cdot (-21) + 3 \cdot 66\right)$$

$$= -5(-147 + 198)$$

$$= -5 \cdot 51 = -255$$

3. *Bonus!* Justify your answer to (1f) above.

$\det(M) = \det(M^T)$

"swapping rows $\Rightarrow$ changing signs"

$$\begin{vmatrix} g & d & a \\ h & e & b \\ i & f & c \end{vmatrix} = \begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix} = -\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -5$$

this suggests how to prove in general that "swapping cols $\Rightarrow$ changing signs" it does NOT just assume it's true!