Name:______________________________

Mathematics 205
Exam I
February 17, 2012

Problem	Possible	Actual
1	15	
2	15	
3	6	
4	8	
5	12	
6	14	
7	15	
8	15	
Total	100	

You must show all work to receive credit.
No electronic devices other than calculators are permitted.
Give exact answers (such as $\ln 5$ or e^2) unless requested otherwise.

1. Suppose $B = \left[\begin{array}{cc|c} 1 & 2 & k \\ 3 & h & 8 \end{array}\right]$.

   (a) What is required of h and k so that the system has no solutions?

   (b) What is required of h and k so that the system has a unique solution?

   (c) What is required of h and k so that the system has infinitely many solutions?

2. (a) What are the properties of a subspace?

(b) Let $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ where } |x_1| \geq |x_3| \right\}$. Is S a subspace of $\mathbb{R}^3$?

3. What are the properties required for a mapping to be a linear transformation?

4. Let $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 10 \end{bmatrix}$. Write basis elements for null$B$ and col$B$.

5. Consider $P = \{a_0 + a_1 x + a_2 x^2, a_i \in \mathbb{R}\}$, the set of all constant, linear and quadratic polynomials over $\mathbb{R}$. We may regard an element of P as a vector in $\mathbb{R}^3$.

For example, $\pi + 3x + x^2$, $4 + 8x$, and $a_0 + a_1 x + a_2 x^2$ have the vectors $\begin{bmatrix} \pi \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$.

(a) Let $y'(t)$ be an element of S and define

$$T(y') = \frac{\int_0^x y'(t)\, dt}{x}.$$

Show that T is a linear transformation.

(b) What is the matrix of the transformation of this map. (Hint: The vector $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ corresponds to polynomial x. Where is x mapped to and what is the vector that corresponds to that answer?)

6. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$.

(a) What does it mean for a mapping to be onto? Is $T(\vec{x}) = A\vec{x}$ onto?

(b) What does it mean for a mapping to be one-to-one? Is $T(\vec{x}) = A\vec{x}$ one-to-one?

7. A matrix A is invertible. Write five equivalent statements from the Invertible Matrix Theorem.

8. Balance the following chemical reaction using techniques learned in class.

$$PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO.$$