Math 205B&C 02/27/09 Quiz 04 (rev) page 1 **Name** Suggested solution **8am** **1:10 pm**

1. Let $A = \begin{bmatrix} 1 & 3 & -5 & 0 & -5 \\ 1 & 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 & 0 \\ 3 & 5 & 1 & 3 & 2 \end{bmatrix}$; then $\text{RREF}(A|I_4) = \left[\begin{array}{ccccc|cccc} 1 & 0 & 7 & 0 & 1 & 1 & 0 & -9 & 3 \\ 0 & 1 & -4 & 0 & -2 & 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 & 3 & -1 & 0 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & -1 \end{array}\right]$

1A. Use the above information to find explicit conditions on $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ that guarantee $\mathbf{b}$ is in Col(A).

The last row of $\text{RREF}(A|I_4)$ tells us that $0 = 0b_1 + 1b_2 + 2b_3 - 1b_4$, or, $b_2 = -2b_3 + b_4$

1B. Show that $\mathbf{d} = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 16 \end{bmatrix}$ satisfies the conditions in (1A).

Does $2 = -2 \cdot 7 + 16$?

$2 = -14 + 16$?

$2 = 2$ ✓

1C. Find a basis for Col(A).

Its pivot columns are cols 1, 2 and 4, so $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$ is a basis.

1D. Express $\mathbf{d}$ as a linear combination of the basis vectors from (1C).

row reduction of $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 6 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 7 \\ 3 & 5 & 3 & 16 \end{array}\right]$ yields $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$, so $\begin{bmatrix} 6 \\ 2 \\ 7 \\ 16 \end{bmatrix} = -9 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} + 5 \begin{bmatrix} 3 \\ 1 \\ 2 \\ 5 \end{bmatrix} + 6 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix}$

1E. Find a basis for Nul(A).

soln's of $A\vec{x} = \vec{0}$ are $x_3 \begin{bmatrix} -7 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 2 \\ 0 \\ -3 \\ 1 \end{bmatrix}$ where x_3 & x_5 are free. ∴ $\left\{ \begin{bmatrix} -7 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$ spans $\{\vec{v}_h\}$; it's a L.I. set so it's A BASIS

1F. Is the first column vector of A in Col(RREF(A))? Explain. (*Hint:* What is the bottom entry of *any* linear combination of the column vectors of RREF(A)?)

$\text{Col}(\text{RREF}(A)) = \left\{ \text{all L.C.'s of } \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix} \right\}$ Any such L.C. is of the form $\begin{bmatrix} * \\ * \\ * \\ 0 \end{bmatrix}$. Since $\vec{c}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$, and $0 \neq 3$, $\vec{c}_1$ can NOT be in Col(RREF(A)).

1G. *Bonus!* Is either of the first two column vectors of RREF(A) in Col(A)? Explain. (*Hint:* See 1A.) Does Col(A) = Col(RREF(A))? Explain!

note that $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ do and do not satisfy $b_2 = -2b_3 + b_4$, respectively. So $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is in, and $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ is NOT in, Col(A).

Since some vectors in Col(RREF(A)) are not members of Col(A) (eg. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$) these 2 sets are UNEQUAL. (note also some members of Col(A) are not members of Col(RREF(A)) - see 1F!

2. Let H be the subset of vectors in $\mathbf{R}^3$ of the form $\begin{bmatrix} a \\ b \\ a^2 + b^2 \end{bmatrix}$, where a and b are any real numbers. Is H closed under vector addition? Prove it or give an explicit counterexample.

note that $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 13 \end{bmatrix}$ belong to H since $1^2 + 2^2 = 5$ and $2^2 + 3^2 = 13$. The sum of these two vectors is $\begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}$ which is NOT in H because $3^2 + 5^2 = 9 + 25 = 34 \neq 18$. So H is not closed under vector addition.