**Math 205B&C** 02/27/09 Quiz 04 (rev) page 1 **Name**____________________ **8am** **1:10 pm**

**1.** Let $A = \begin{bmatrix} 1 & 3 & -5 & 0 & -5 \\ 1 & 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 & 0 \\ 3 & 5 & 1 & 3 & 2 \end{bmatrix}$; then $\text{RREF}(A|I_4) = \left[\begin{array}{ccccc|cccc} 1 & 0 & 7 & 0 & 1 & 1 & 0 & -9 & 3 \\ 0 & 1 & -4 & 0 & -2 & 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 & 3 & -1 & 0 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & -1 \end{array}\right]$

**1A.** Use the above information to find explicit conditions on $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ that guarantee $\mathbf{b}$ is in Col($A$).

**1B.** Show that $\mathbf{d} = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 16 \end{bmatrix}$ satisfies the conditions in (1A).

**1C.** Find a basis for Col($A$).

**1D.** Express $\mathbf{d}$ as a linear combination of the basis vectors from (1C).

**1E.** Find a basis for Nul($A$).

**1F.** Is the first column vector of $A$ in Col(RREF($A$))? Explain. (*Hint:* What is the bottom entry of *any* linear combination of the column vectors of RREF($A$)?)

**1G.** *Bonus!* Is either of the first two column vectors of RREF($A$) in Col($A$)? Explain. (*Hint:* See 1A.) Does Col($A$) = Col(RREF($A$))? Explain!

**2.** Let $H$ be the subset of vectors in $\mathbf{R}^3$ of the form $\begin{bmatrix} a \\ b \\ a^2 + b^2 \end{bmatrix}$, where $a$ and $b$ are any real numbers. Is $H$ closed under vector addition? Prove it or give an explicit counterexample.