

Math 205 Section B
Test 2 (50 points)

Name: _____

- Check that you have 6 questions on three pages.
- Show all your work to receive full credit for a problem.

1. **(10 points)** Let A and B be 4×4 matrices, with $\det A = -10$ and $\det B = 6$. Use properties of determinants to compute:

(a) $\det A^2 B^T$

(b) $\det 2B$

(c) $\det C$ where C is obtained from A by interchanging rows 2 and 4

(d) $\det (B^2)^{-1}$ if B^2 is invertible. Otherwise, explain why B^2 is not invertible.

2. (8 points) Let $A = \begin{bmatrix} 2 & 4 & -2 \\ 6 & 7 & -3 \end{bmatrix}$.

(a) Find an LU factorization of A .

(b) Use the LU factorization in part (a) to solve the equation $A\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

3. (8 points) Let $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$. Define a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $T(\vec{x}) = A^T \vec{x}$.

(a) Write a formula for $T(x_1, x_2)$.

(b) Is T invertible? Explain.

4. **(8 points)** Suppose a 5×5 matrix A is invertible. Explain (using pivots) why the columns of A are linearly independent. (If you find it helpful, you may use the following steps in your explanation.)

Since A is invertible, A reduces to _____.

So the number of pivots in A is _____.

Use this to explain why the columns of A are linearly independent.

5. (8 points) Determine if each of the following sets is a subspace of the appropriate vector space.

(a) Let $W = \left\{ \begin{bmatrix} a \\ b \\ a + b \end{bmatrix} : a, b \text{ are real numbers.} \right\}$. Is W a subspace of \mathbb{R}^3 ? Explain.

(b) Let W be the set of all polynomials of the form $p(t) = t + a$ where a is a real number. Is W a subspace of \mathbb{P}_1 ? Explain. (\mathbb{P}_1 is the set of all polynomials of degree at most one.)

6. (8 points) Suppose $AB = \begin{bmatrix} 3 & -1 \\ 1 & 0 \\ -2 & 5 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$.

(a) Find the inverse of A , if it exists.

(b) Find B . (You may use your answer in part (a) to answer this question.)