MATH 106B - CALCULUS II
WINTER 2006

QUIZ 6

NAME:

Show ALL your work CAREFULLY.

(a) Find the third-order Maclaurin polynomial $M_3(x)$ for the function $f(x) = \sin^2 x$. [Hint: you may want to use the following identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x).$

Let $f(x) = \sin^2 x$. Using the trig. identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, we get $f'(x) = 2\sin x \cos x$, $f''(x) = 2\cos 2x$, and $f'''(x) = -4\sin 2x$. Note that $f(0) = f'(0) = f''(0) = 0$ while $f'''(0) = 2$. Thus,

$$M_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 = x^2.$$ 

(b) Find the third-order Taylor polynomial $P_3(x)$ for $f(x)$ centered at $x_0 = \frac{\pi}{2}$.

Now,

$$P_3(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3$$

$$= 1 + 0 \cdot \left(x - \frac{\pi}{2}\right) + \frac{(-2)}{2!} \left(x - \frac{\pi}{2}\right)^2 + 0 \cdot \left(x - \frac{\pi}{2}\right)^3$$

$$= 1 - \left(x - \frac{\pi}{2}\right)^2.$$ 

Date: February 17, 2006.