(12) I. Suppose \( f(x, y) = x \sin(xy) \)

A. \( \frac{\partial f}{\partial x} (x, y) = \)

B. \( \frac{\partial f}{\partial y} (x, y) = \)

C. \( \frac{\partial^2 f}{\partial x \partial y} (x, y) = \)

D. \( \frac{\partial^2 f}{\partial y \partial x} (x, y) = \)
II. Suppose \( f : \mathbb{R}^3 \to \mathbb{R}^3 \) with rule \( f(x, y, z) = (xyz, xy, x) \).

A. Calculate \( Jf(2, 2, 2) \), the Jacobian matrix of \( f \) at \( (2, 2, 2) \).

B. Find a point at which \( Df(2, 2, 2) \), the total derivative of \( f \), has the value \( (0, 8, 2) \).
(12) III. For the function \( f(x, y, z) = x + y \) at the point \((1, 1, 0)\) compute:

A) The Hessian matrix.

B) The Hessian form.

C) The second-degree Taylor polynomial.
(12) IV. Suppose \( F : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) with rule \( F(x, y, z) = (x^2, y^2, x^2 - y^2) \) and \( G : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) with rule \( G(x, y, z) = (x^2 + y^2 + z^2, x + y + z, z) \).

A. Calculate the Jacobian matrix of \( F \) at the point \((1, 2, 1)\).

B. Calculate the Jacobian matrix of the function \( G \) at the point \( F(1, 2, 1) \).

C. Calculate the Jacobian matrix of the function \( G \circ F \) at the point \((1, 2, 1)\).
(10) V. Find the equation of the tangent plane at the point \((0, 1, 1)\) to the surface with equation:

\[ x^3 - 5y^2 + 6yz^3 = 1. \]

(10) VI. Suppose \(f(x, y, z) = xy^2 + x^2y - z - 5x\) and \(a = (1, 1, 1)\).

Compute the directional derivative of \(f\) at \(a\) in the direction parallel to the line \(x(t) = (t + 1, t + 2, t + 3).\)
(10) VII. Find all critical points of $f(x, y) = x^2 + y^2 - 4x - 2y + 5$. Use the Second Derivative Test to determine whether each critical point is a local minimum, a local maximum, or neither.
(8) VIII. Suppose \( f(x, y) = \frac{x^2 - y^2}{x^2 + y^2} \).

A. What is the domain of \( f \)?

B. \( \lim_{(x, y) \to (0, 0)} f(x, y) \) does not exist, and is so nasty that given any number you pick from \(-1 \) to \( 1 \) you can find a line along which to approach \((0, 0)\) and get your number. Find a value for \( k \) so that if you approach \((0, 0)\) along the line \( y = kx \) the limit of \( f(x, y) \) will be \( 4/5 \).
(8) IX Give an example [a sketch is sufficient] of:

A. An open set in \( \mathbb{R}^2 \) that is not bounded.

B. A closed set in \( \mathbb{R}^2 \) that is bounded.
Suppose \( f(x, y) = \sqrt{x^2 - y^2} \)

A. What is the domain of \( f \)?

B. What is \( \nabla f(x, y) \)?

C. Find all the points where the gradient of \( f \) is zero.

D. Find all the points where the gradient is undefined. [This is an infinite set.]

E. Find the minimum value of \( f \) and the values of \( x \) and \( y \) where it occurs.