Math 205A Quiz 05 page 1 February 15, 2008 NAME suggested sol'n's.

1. Let $A = \begin{bmatrix} 3 & 6 & 0 & 4 \\ 6 & s & 2 & t \\ 1 & 5 & 3 & 8 \\ 2 & u & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 0 & 7 \\ 1 & 7 & 3 & s \\ 2 & 2 & 9 & t \\ 1 & 4 & 1 & 1 \end{bmatrix}$. Let $C = AB$ and suppose $C = \begin{bmatrix} ? & ? & h & ? \\ ? & 28 & ? & ? \\ 23 & ? & ? & 29 \\ ? & ? & 24 & ? \end{bmatrix}$.

1A. Find h.

$$h = \left(\text{row 1 of } A\right) * \left(\text{col 3 of } B\right) = \begin{bmatrix} 3 & 6 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 9 \\ 1 \end{bmatrix} = 3\cdot 0 + 6\cdot 3 + 0\cdot 9 + 4\cdot 1 = 22$$

1B. Find u.

note that $24 = \left(\text{row 4 of } A\right) * \left(\text{col 3 of } B\right) = \begin{bmatrix} 2 & u & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 9 \\ 1 \end{bmatrix} = 2\cdot 0 + u\cdot 3 + 1\cdot 9 + 0\cdot 1$

$= 3u + 9$

so $3u + 9 = 24 \Rightarrow u = 5$

1C. Find s.

note that $\begin{bmatrix} 6 & s & 2 & t \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \\ 4 \end{bmatrix} = 28$

AND $\begin{bmatrix} 1 & 5 & 3 & 8 \end{bmatrix} \begin{bmatrix} 7 \\ s \\ t \\ 1 \end{bmatrix} = 29$

so $\begin{cases} 6 + 7s + 4 + 4t = 28 \\ 7 + 5s + 3t + 8 = 29 \end{cases} \Leftrightarrow \begin{cases} 7s + 4t = 18 \\ 5s + 3t = 14 \end{cases}$

1D. Find t.

now, $\left[\begin{array}{cc|c} 7 & 4 & 18 \\ 5 & 3 & 14 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 8 \end{array}\right] \Rightarrow \begin{matrix} s = -2 \\ t = 8 \end{matrix}$

2. Let $D = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$.

2A. Find D^{-1}.

$$\frac{1}{7\cdot 3 - 5\cdot 4} \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix} = 1 \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$$ or, simply $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

2B. Use D^{-1} to solve $\underbrace{D \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 14 \end{bmatrix}}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = D^{-1} \begin{bmatrix} 18 \\ 14 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} 18 \\ 14 \end{bmatrix} = \begin{bmatrix} 3\cdot 18 - 4\cdot 14 \\ -5\cdot 18 + 7\cdot 14 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

(note this is the same problem encountered in 1C & 1D above)

3. Suppose that E, Q, P and G all belong to $M_{3\times 3}$.
Under what conditions does $(GQPE)^{-1}$ exist, and what is $(GQPE)^{-1}$ in this case?

$(GQPE)^{-1}$ exists $\Leftrightarrow$ each of E^{-1}, Q^{-1}, P^{-1}, & G^{-1} exists,

in which case $(GQPE)^{-1}$ is $E^{-1} \cdot P^{-1} \cdot Q^{-1} \cdot G^{-1}$