Exam #1, Math 205A (Linear Algebra)

This take-home exam is due at class time on **Friday, February 15**. (Sooner is fine.) You may consult the textbook (or any other book) and any class notes and handouts, but **please do not discuss any details of this exam with anyone except me!** Please sign the bottom of this sheet and turn it in with your exam. You may ask me questions about the exam, but I reserve the right to give unsatisfying answers. Please show all work (though you are encouraged to *check* your answers on MATLAB or a calculator).

1. (8 points) Find the angle between the vectors $\begin{pmatrix} 3 \\ 0 \\ 4 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 4 \\ 4 \\ 1 \end{pmatrix}$.

2. (18 points) Solve the system $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 7 \\ 19 \end{pmatrix}$ by any method (by hand).

3. (20 points) Find A^{-1} (by any method, but by hand) if $A = \begin{pmatrix} 2 & 0 & 0 & 3 \\ 0 & 3 & 4 & 0 \\ 0 & 4 & 5 & 0 \\ 3 & 0 & 0 & 4 \end{pmatrix}$.

4. (30 points) (a) Find the LU factorization of $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}$. (This is the matrix pascal(4) that some of you were looking at on MATLAB the other day.)

(b) Use your answer to (a) to solve $A\vec{x} = \begin{pmatrix} 1 \\ 8 \\ 27 \\ 64 \end{pmatrix}$.

(c) Find L^{-1} and U^{-1}. Please show work for at least one of these.

(d) Use your answers to (c) to compute A^{-1}.

5. (14 points) Let R be the 2×2 matrix that reflects vectors in $\mathbb{R}^2$ in the line making angle ϕ with the positive x-axis, and let P be the 2×2 matrix that projects vectors in $\mathbb{R}^2$ onto the same line. The formulas for R and P are

$$R = \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} \cos^2 \phi & \cos\phi \sin\phi \\ \cos\phi \sin\phi & \sin^2 \phi \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \end{pmatrix}$$

What is the product RP? Try to find a nice expression for it. Is it the same as PR, or are they different? Try to explain your answers geometrically.

6. (10 points) Suppose that A is a 2×2 matrix which commutes with its transpose (*i.e.*, $AA^T = A^T A$). Show that either $A = A^T$ or A is a multiple of a rotation matrix.

I affirm that I did not receive help from another person in doing this exam, nor did I give help to another student in the class.

(signed) _____