

## Math 205A Test 1 (50 points)

Name: \_\_\_\_\_

- Check that you have 8 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (5 points) (For this problem do all calculations by hand.) The augmented matrix of a system of equations is given below. Find all possible value(s) of  $h$  so that the system has a solution.

$$\left[ \begin{array}{ccc} 1 & h & 2 \\ 2 & 5 & 4 \end{array} \right]$$

2. (5 points) Suppose  $A$ ,  $B$ ,  $C$  and  $X$  are invertible  $n \times n$  matrices and  $A(X + B)^T = CA$ . Solve for  $X$ .

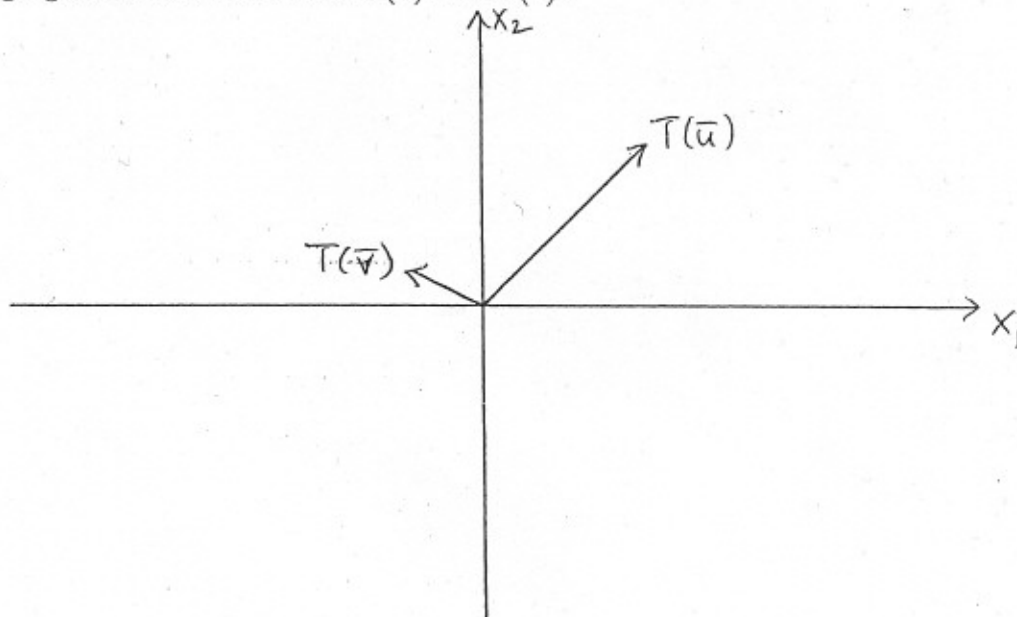
3. (6 points) Let  $A$  be a matrix such that its columns are vectors in  $\mathbb{R}^6$ . The general solution of the equation  $A\vec{x} = \vec{0}$  is as follows.

$$\begin{aligned}x_1 &= 2x_3 - x_5 \\x_2 &= x_3 + x_5 \\x_4 &= 3x_5 \\x_3, x_5 &\text{ are free}\end{aligned}$$

- (a) Are the columns of  $A$  linearly independent? Explain.

- (b) Do the columns of  $A$  span  $\mathbb{R}^6$ ? Explain.

4. (5 points) Suppose  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a linear transformation and  $\vec{u}$  and  $\vec{v}$  are two vectors in  $\mathbb{R}^2$ . The following figure shows the vectors  $T(\vec{u})$  and  $T(\vec{v})$ .



- (a) If  $\vec{w} = \vec{u} - 2\vec{v}$ , draw  $T(\vec{w})$  in the above figure. Show clearly how you draw  $T(\vec{w})$  and state clearly what you know about  $T(\vec{w})$ .

- (b) Is  $\{T(\vec{u}), T(\vec{v})\}$  a linearly independent set? Explain.

5. (10 points) Let  $A = \begin{bmatrix} -6 & -3 \\ 11 & 5.5 \\ 2 & 1 \end{bmatrix}$ . Let  $T$  be a linear transformation given by  $T(\vec{x}) = A\vec{x}$

(a) Describe the vectors in  $\text{Nul } A$  in parametric vector form.

(b) Is  $T$  one-to-one? Explain.

(c) Find three distinct non-zero vectors in  $\text{Col } A$ .

6. (5 points) Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a transformation such that

$$T(x_1, x_2, x_3) = (9x_3 - x_1, 4x_2x_3 + x_1).$$

Is  $T$  a linear transformation? Explain.

7. (5 points) Let  $H = \left\{ \left[ \begin{array}{c} b + d \\ a - d \\ a - 2c + d \\ c + 3d \end{array} \right] : a, b, c, d \text{ are real numbers.} \right\}$ .

Is  $H$  a subspace of  $\mathbb{R}^4$ ? Explain.

8. (9 points) A mining company has three mines. One day of operation at the mines produces the following output.

Mine 1 produces 20 tons of copper, 600 kilograms of silver and 10 tons of manganese.

Mine 2 produces 30 tons of copper, 500 kilograms of silver and 14 tons of manganese.

Mine 3 produces 25 tons of copper, 550 kilograms of silver and 12 tons of manganese.

Suppose the company has orders for 545 tons of copper, 11350 kilograms of silver and 260 tons of manganese.

(a) Write a vector equation to answer the question: how many days should the company operate each mine to exactly fill the orders? State clearly what the variables represent.

(b) Give two possible answers to the question in part (a). In how many ways can we answer the question in part(a)? Explain.