NAME:

Show ALL your work CAREFULLY.
[Just in case, \( \int \sec x \, dx = \ln | \sec x + \tan x | + C. \)]

Use trigonometric substitution to find each of the following indefinite integrals.

(a) \[ \int \frac{dx}{\sqrt{4x - x^2 - 3}} \]

Note that \( \sqrt{4x - x^2 - 3} = \sqrt{1 - (x^2 - 4x + 4)} = \sqrt{1 - (x - 2)^2} \). By letting \( x - 2 = \sin t \), we have \( dx = \cos t \, dt \) and so \( \sqrt{4x - x^2 - 3} = \cos t \). Thus,

\[
\int \frac{dx}{\sqrt{4x - x^2 - 3}} = \int \frac{\cos t \, dt}{\cos t} = t + C = \arcsin(x - 2) + C.
\]

(b) \[ \int \frac{dx}{\sqrt{2 + x^2}} \]

Let \( x = \sqrt{2} \tan \theta \) so that \( dx = \sqrt{2} \sec^2 \theta \, d\theta \). It follows that

\[
\int \frac{dx}{\sqrt{2 + x^2}} = \int \frac{\sqrt{2} \sec^2 \theta \, d\theta}{\sqrt{2} \sec \theta} = \int \sec \theta \, d\theta = \ln | \sec \theta + \tan \theta | + C.
\]

Thus,

\[
\int \frac{dx}{\sqrt{2 + x^2}} = \ln \left| \frac{x^2}{2} + 1 + \frac{x}{\sqrt{2}} \right| + C.
\]

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