1. Let $A = \begin{bmatrix} 2 & 3 & 5 & -1 \\ 4 & 3 & 7 & 7 \\ 3 & 2 & 5 & 6 \end{bmatrix}$, let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ and let $\mathbf{c} = \begin{bmatrix} 15 \\ -3 \\ -5 \end{bmatrix}$.

1A. Find all solutions of $A\mathbf{x} = \mathbf{c}$ and write your answer in parametric vector form $\mathbf{x} = \mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{c}$ and and $\mathbf{v}_h$ represents all solutions of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

1B. Find any/all conditions on b_1, b_2 and b_3 which are necessary and sufficient for $\mathbf{b}$ to be in the span of the set of column vectors of A.

1C. Give the correct definition: What does it mean to say a set of vectors $S = \{\mathbf{v}_1, \ \mathbf{v}_2, \ \ldots, \ \mathbf{v}_p\}$ is *linearly independent*?

"We say S is linearly independent if and only if..." [you complete the definition:]

1D. Do the column vectors of A form a linearly independent set? Explain in terms of the definition of linear independence.

1E. Do the column vectors of A span $\mathbb{R}^3$? Explain.

2. Suppose $T : \mathbb{R}^k \to \mathbb{R}^q$ is a transformation. Define what it means for T to be a *linear* transformation.

3. Suppose $T\left(\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}\right) = \begin{bmatrix} {s_1}^2 - {s_2}^2 \\ s_1 + s_2 + s_3 \end{bmatrix}$ is a transformation from $\mathbb{R}^3 \to \mathbb{R}^2$.

Show T is not a linear transformation and in fact fails both parts of the definition in problem (2). In your counterexamples' vectors, use all-different, positive, single digit numbers for the s_i's.

4. Suppose A is a matrix with six column vectors $\mathbf{c}_1, \ldots, \mathbf{c}_6$.

Suppose all the solutions to $A\mathbf{x} = \mathbf{0}$ are given by $\mathbf{v}_h = x_3 \begin{bmatrix} -6 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 7 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, where x_3, x_5 and x_6 are free.

4A. Express $\mathbf{0}$ as a linear combination of the six columns of A in such a way that in the LC the weights of column vectors three, five and six are 10, 20 and 30, respectively.

4B. Is it possible to find some way to express the first column vector of A as a linear combination of column vectors two through six? If so, do it. If not, explain why this is impossible.

4C. Is it possible to find some way to express the fourth column vector of A as a linear combination of the other column vectors? If so, do it. If not, explain why this is impossible.

4D. *Bonus!* Find an actual matrix A which has the above solutions to $A\mathbf{x} = \mathbf{0}$ and such that the system of eqn's represented by $A\mathbf{x} = \mathbf{b}$ is *always* consistent.

5. Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^4$ is a *linear* transformation and $T\left(\begin{bmatrix}6\\3\\1\end{bmatrix}\right) = \begin{bmatrix}12\\4\\5\\12\end{bmatrix}$ and $T\left(\begin{bmatrix}8\\4\\1\end{bmatrix}\right) = \begin{bmatrix}16\\5\\6\\16\end{bmatrix}$.

5A. Find $T\left(10\begin{bmatrix}6\\3\\1\end{bmatrix} + \begin{bmatrix}8\\4\\1\end{bmatrix}\right)$.

5B. Find $T\left(\begin{bmatrix}4\\2\\1\end{bmatrix}\right)$.

5C. Let D be the standard matrix of T. How many rows and columns, respectively, does D have?

Number of ROWS: Number of COLUMNS:

5D. Find the last column of D, or explain why you can't.

5E. Find the first column of D, or explain why you can't.

6. For each separate problem below, find a matrix A in RREF (reduced row echelon form) with exactly 3 rows that satisfies the conditions given in the problem. Create your RREF using as *few* zeros as possible. If there is no such RREF, explain why.

6A. $A\mathbf{x} = \mathbf{b}$ has exactly one solution for every $\mathbf{b}$ in $\mathbb{R}^3$.

6B. A is the standard matrix of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ and T is onto $\mathbb{R}^3$.

6C. A has 5 columns and some vectors in $\mathbb{R}^3$ are not in the span of those five column vectors.

6D. A is the standard matrix of a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ and T is one-to-one.