

Math 205A Test 1 (50 points)

Name: _____

- Check that you have 7 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (8 points) Let $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3]$ be a 3×3 matrix. Suppose $\vec{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ is a solution of the equation $A\vec{x} = \vec{0}$.

(a) Are the columns of A linearly independent? Explain. If the columns are not linearly independent, express one of the columns as a linear combination of the other columns.

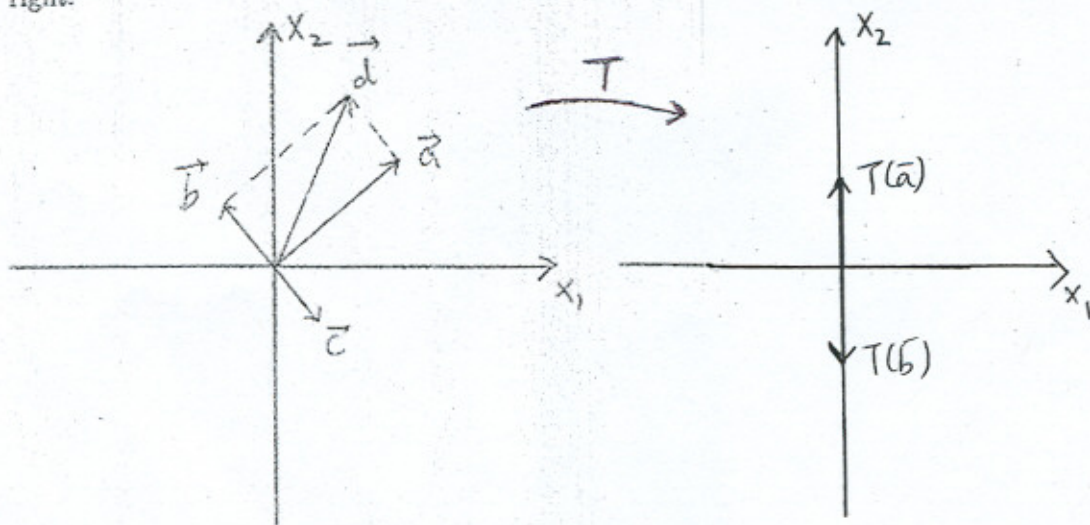
(b) Is A invertible? Explain.

2. (8 points) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ -7 \\ 0 \\ 1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 6 \end{bmatrix}$. Let $W = \text{Span} \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(a) Is $\vec{v} = \begin{bmatrix} 0 \\ -3 \\ 0 \\ 4 \end{bmatrix}$ in W ? Explain.

(b) We know W is a subspace of \mathbb{R}^4 . Does $W = \mathbb{R}^4$? Explain.

3. (8 points) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation. The vectors \vec{a} , \vec{b} , \vec{c} and \vec{d} in \mathbb{R}^2 are shown in the figure on the left. The vectors $T(\vec{a})$ and $T(\vec{b})$ are shown in the figure on the right.



(a) Draw $T(\vec{c})$ and $T(\vec{d})$ in the figure on the right.

(b) Is T one-to-one? Explain.

4. (8 points) Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 - x_3, 3x_3 + 6x_4, x_3 - x_1 - 2x_2).$$

(a) Find the standard matrix of T .

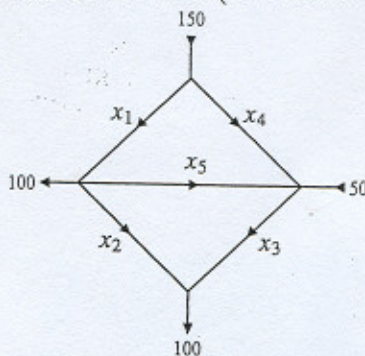
(b) Is T onto? Explain.

5. (5 points) Suppose A , B , C and X are invertible $n \times n$ matrices and $AX + B = CX$. Solve for X . If you need to invert a matrix, explain why that matrix is invertible.

6. (5 points) Let $H = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ where } x_1 < 0, x_2 < 0, x_3 > 0 \text{ and } x_1, x_2, x_3 \text{ are real numbers.} \right\}$.

Is H a subspace of \mathbb{R}^3 ? Explain.

7. (8 points) The figure below shows the traffic flow (in vehicles per half-hour) over some streets in a city.



- (a) Write a system of equations that describes the traffic flow in the above network. Remember that at each intersection the traffic flow in must equal the traffic flow out.
- (b) Write the general solution of the system you wrote in part (a) in parametric vector form. Based on the general solution, describe the flow of traffic (by giving values of the various variables) when the road with flow x_4 is closed and the road with flow x_5 has a flow of 20 vehicles per half-hour.