MATH 205 (B, C) Winter 2009 Review Questions for Exam I

① Define $T: \mathbb{R}^2 \to \mathbb{R}^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 x_2 \\ 5x_1 - 3x_2 \\ x_1 + 2x_2 + 3 \end{bmatrix}$ Show T is NOT a Linear transformation and in fact it fails BOTH parts ① & ② of our L.T. defn.

② Define $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} 11a + 33b + 20c + 144d \\ 5a + 15b + 9c + 65d \\ 3a + 9b + 2c + 22d \end{bmatrix}$

ⓐ $T: \mathbb{R}^{\boxed{?}} \to \mathbb{R}^{\boxed{?}}$ ⓑ find $T\left(\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}\right)$, $T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}\right)$, $T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right)$

ⓒ what matrix gives $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right)$? Call it A.

ⓓ Is $\begin{bmatrix} 31 \\ 14 \\ 5 \end{bmatrix}$ in the image of T? If so, find all $\vec{w}$ for which $T(\vec{w}) = \begin{bmatrix} 31 \\ 14 \\ 5 \end{bmatrix}$ and express your answer in the "$\vec{p} + \vec{v}_h$" form where $\vec{p}$ is a particular soln & $\vec{v}_h$ is all solns of $T(\vec{x}) = \begin{bmatrix} 31 \\ 14 \\ 5 \end{bmatrix}$ and $T(\vec{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ respectively.

ⓔ repeat ⓓ for $\begin{bmatrix} 31 \\ 14 \\ 15 \end{bmatrix}$

ⓕ are there general conditions which $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ must satisfy in order to be in the image of T?

ⓖ Is T onto? explain!

③ Let B be this matrix →

$$\begin{bmatrix} 6 & 2 & 0 & 7 & -4 & 0 \\ -13 & -3 & -5 & -4 & 6 & -1 \\ 16 & 4 & 5 & 7 & -8 & 1 \\ 23 & 5 & 10 & 9 & -10 & 2 \end{bmatrix}$$

Label the columns $\vec{c}_1, \cdots, \vec{c}_6$.

ⓐ express all solns of $B\vec{x} = \vec{0}$ in the form $\vec{v}_h$ as done in class.

ⓑ which columns can be expressed as L.C.'s of the others? For each such column vector, do so with as few of the other columns as possible.

ⓒ Is $\{\vec{c}_1, \vec{c}_2\}$ a L.I. set? Explain! ⓓ Is $\{\vec{c}_1, \vec{c}_2, \vec{c}_3, \vec{c}_4\}$ L.I.? (hint: you already did enough row-reduction)

ⓓ Find conditions on $\vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_4 \end{bmatrix}$ which guarantee it to be in the span of the cols of B.

ⓔ verify that $\begin{bmatrix} 17 \\ -37 \\ 45 \\ 70 \end{bmatrix}$ satisfies these conditions. Find a way to express this vector using a L.C. of the columns of B in which NONE of the weights are 0.

ⓕ without actually computing $10\vec{c}_1 + 20\vec{c}_2 + 30\vec{c}_3 + 40\vec{c}_4$, express it as a L.C. of $\vec{c}_1, \vec{c}_3, \vec{c}_4, \vec{c}_5, \vec{c}_6$ or explain why you cannot. Repeat, expressing it as a L.C. of all columns except $\vec{c}_4$.

(4) Suppose C is a matrix for which all solutions of $C\vec{x} = \vec{0}$ have the form $\vec{x} = x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

where x_2 x_3 & x_6 are free; suppose also that the system underlying the matrix equation $C\vec{x} = \vec{b}$ is always consistent (for any $\vec{b}$).
What is the RREF of C? what are the pivot columns?
which columns can/cannot be written as L.C.'s of the others? Fully explain.

(5) Let $T: \mathbb{R}^{\boxed{?}} \to \mathbb{R}^{\boxed{?}}$ have standard matrix $D = \begin{bmatrix} 17 & 2 & -2 \\ 14 & 2 & -3 \\ -9 & -1 & 4 \\ 8 & 1 & -2 \end{bmatrix}$

(a) do the columns of D form a L.I. set?
(b) in "$\vec{V}_h$ form", express all solutions of $D\vec{x} = \vec{0}$.
(c) Find all solutions of $T(\vec{x}) = \vec{0}$
(d) Is the L.T. T onto? explain, or give conditions on $\vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_4 \end{bmatrix}$ that it satisfies to NOT be in the image of T.

(6) Know the defn's of LI, LD, LC, span, LT.

* EXAM I COVERS 1.1 — 1.9 (except 1.6) & 2.1 — 2.3 *

Review Session FEB 12, 2009, 8 pm, Hatham 104

(7a) If A is invertible, what is $(A^{-1})^{-1}$??

(7b) Suppose A is a matrix and applying the row operations represented by the elementary matrices:

$$E_1 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}, \quad E_3 \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

to A in the order "first E_1; then E_2; lastly, E_3" produces I.
what is A? what is A^{-1}?
Does $E_3E_2E_1 = E_1E_2E_3$? is this important?