Math 205 (Winter 2011)

Test 1 (50 points)

Name: ____

- Check that you have 7 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (8 points) Hurricanes develop low pressure at their centers that generates high winds. The maximum wind speed s (in knots) and the central pressure p of a hurricane are approximately related by the equation $a + bp = s$. We have the following data on four recent Atlantic hurricanes in the United States.

p	905	920	960	990
s	130	110	80	60

   (a) Use the data to write a linear system of four equations which might be used to determine a and b.

   (b) Is the system you wrote in part (a) consistent? Explain.

2. (9 points) Suppose B is a 4×4 matrix with columns $\vec{b}_1$, $\vec{b}_2$, $\vec{b}_3$, and $\vec{b}_4$. The solution of the equation $B\vec{x} = \vec{0}$ is given below in parametric vector form.

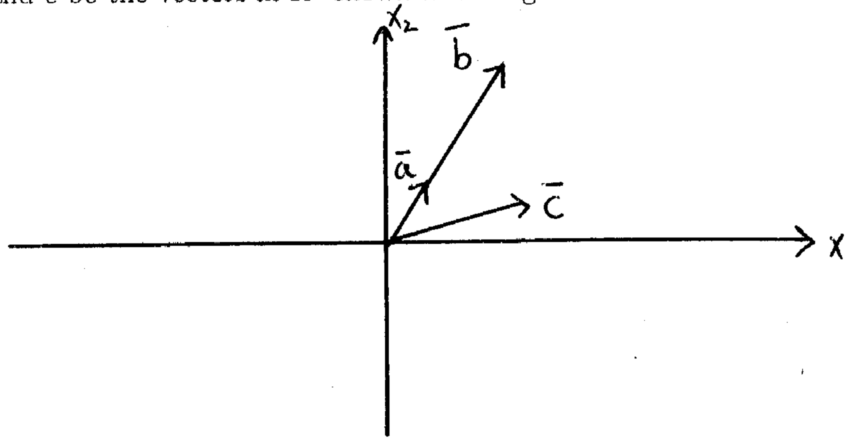
$$\vec{x} = x_2 \begin{bmatrix} -1.5 \\ 1 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0.5 \\ 0 \\ 5 \\ 1 \end{bmatrix}.$$

(a) Is $\vec{b}_1$ in Span $\{\vec{b}_2, \vec{b}_3, \vec{b}_4\}$? Explain.

(b) Suppose $\vec{b}$ is a vector in $\mathbb{R}^4$ such that the equation $B\vec{x} = \vec{b}$ is consistent. How many solutions does the equation have? Explain.

(c) Is B invertible? Explain.

3. (8 points) Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be the vectors in $\mathbb{R}^2$ shown in the figure.



(a) Give a geometric description of Span$\{\vec{b}\}$.

(b) Is the set $\{\vec{a}, \vec{c}\}$ linearly independent? Explain.

(c) Is the set $\{\vec{a}, \vec{b}\}$ linearly independent? Explain.

(d) Write a non-trivial solution of the vector equation $x_1\vec{a} + x_2\vec{b} + x_3\vec{c} = \vec{0}$.

4. (5 points) Suppose T is a transformation given by the formula $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 + x_2 \\ x_2 \\ x_1 - x_2 \end{bmatrix}$.

   (a) What are the domain and codomain of T?

   (b) Show that T is not a linear transformation by providing a counterexample.

5. (4 points) Suppose the second column of a matrix B is twice the first column and the sum of the first three columns of B is the zero vector. Let A be a matrix such that the product AB is defined.

   (a) Show that the second column of AB is twice the first column of AB.

   (b) Show that the sum of the first three columns of AB is the zero vector.

6. (9 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(\vec{e}_1) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ and $T(\vec{e}_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(a) Find the standard matrix of T.

(b) Find $T(\vec{v})$ where $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ 7 \end{bmatrix}$.

(c) Is T onto? Explain.

7. (5 points) Let $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$, and $\vec{a}_3 = \begin{bmatrix} 2 \\ h \\ 9 \end{bmatrix}$.

(a) Let $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3]$, i.e., A is the matrix with $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$ as its columns. Find the row echelon form (REF) (**NOT** RREF) of A. Show all the calculations by hand.

(b) Find all possible value(s) of h such that the set $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ is linearly independent.