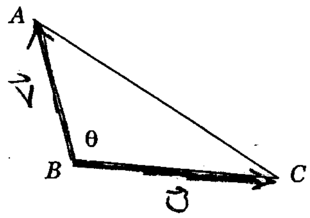
Name: Solutions

Math 206A: Winter 2012
Exam 1: February 10

Correct answers accompanied by incorrect or incomplete work will not receive full credit.

Good Luck!

1. Consider the triangle with vertices $A = (1, 1, 1)$, $B = (3, -2, 3)$, and $C = (3, 4, 6)$. (Figure may not be drawn to scale.)



(a) (10 points) Find $\angle ABC$, i.e., find θ. In your work use correct vector notation.

$$\vec{u} = \vec{BC} = (3,4,6) - (3,-2,3) = (0,6,3) \qquad \|\vec{u}\| = \sqrt{36+9} = \sqrt{45}$$

$$\vec{v} = \vec{BA} = (1,1,1) - (3,-2,3) = (-2,3,-2) \qquad \|\vec{v}\| = \sqrt{4+9+4} = \sqrt{17}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{0+18+{}^-6}{\sqrt{45}\,\sqrt{17}} \approx .4339$$

$$\theta \approx \cos^{-1}(.4339) = 1.12 \text{ rad} = 64.3^\circ$$

or

(b) (10 points) Find the area of the triangle. In your work use correct vector notation.

$$\text{Area} = \frac{1}{2}\|\vec{v} \times \vec{u}\|$$

$$\vec{v}\times\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & -2 \\ 0 & 6 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -2 \\ 6 & 3 \end{vmatrix}\hat{i} - \begin{vmatrix} -2 & -2 \\ 0 & 3 \end{vmatrix}\hat{j} + \begin{vmatrix} -2 & 3 \\ 0 & 6 \end{vmatrix}\hat{k}$$

$$= (9 - {}^-12)\hat{i} - (-6-0)\hat{j} + (-12-0)\hat{k} = (21)\hat{i} + 6\hat{j} - 12\hat{k}$$

$$\|\vec{v}\times\vec{u}\| = \sqrt{21^2 + 36 + 144} = \sqrt{621}$$

$$\text{Area} = \boxed{\frac{1}{2}\sqrt{621}} \approx 12.46$$

2. Consider the equation $Ax^2 + By^2 + Cz^2 = D$. Fill in each blank with a **single** number that makes the statement correct.

(a) (10 points) If $A =$ __1__, $B =$ __-1__, $C =$ __-1__, and $D =$ __1__ then the graph of the equation is a hyperboloid of 2 sheets with axis being the x-axis.

$\hookrightarrow$ 2 of A, B, C must be neg and $D > 0$

pos coeff corresponds to axis so $A > 0$

(other answers are possible)

(b) (10 points) If $A =$ __1__, $B =$ __-1__, $C =$ __1__, and $D =$ __0__ then the graph of the equation is a (double) cone with axis being the y-axis.

$\hookrightarrow$ $y^2 = x^2 + z^2$ is such a cone.

$\Rightarrow$ $x^2 - y^2 + z^2 = 0$

(other answers are possible)

3. (10 points) Write the equation of the ellipsoid with center $(2, -3, 5)$ that is tangent to the planes $x = 0$, $y = 0$, and $z = 0$.

x-semiaxis $= 2$ b/c ctr is 2 units from $x=0$ plane

y-semiaxis $= 3$ b/c ctr is 3 units from $y=0$ plane

z-semiaxis $= 5$ b/c ctr is 5 units from $z=0$ plane.

So eqn is

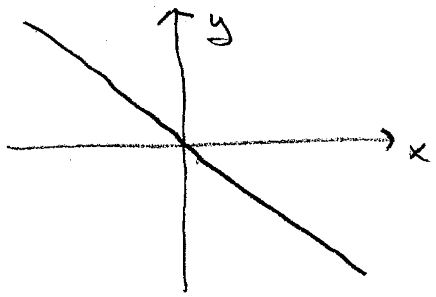
$$\frac{(x-2)^2}{2^2} + \frac{(y+3)^2}{3^2} + \frac{(z-5)^2}{5^2} = 1.$$

4. (a) (15 points) Sketch the $z = 0$, $x = 0$, and $y = 0$ traces of $f(x, y) = |x + y|$.

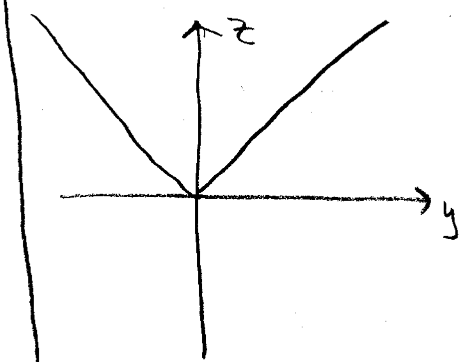
$z=0$; $0=|x+y|$

$0=x+y$

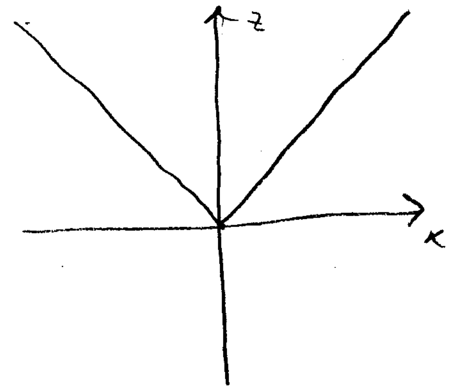
$-x=y$



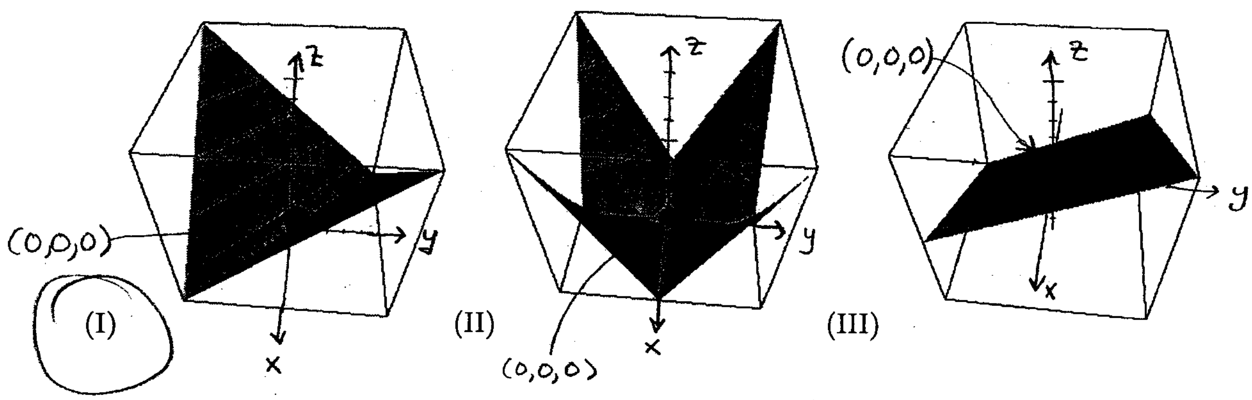
$x=0$; $z=|y|$



$y=0$; $z=|x|$

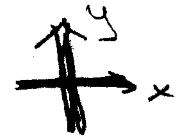


(b) (5 points) Which of the following graphs is the graph of $f(x, y) = |x + y|$?



not II b/c it has the wrong $z=0$ trace

bottom of "box" intersects graph

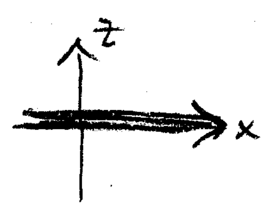


not III b/c it has wrong $x=0$ & $y=0$ traces.

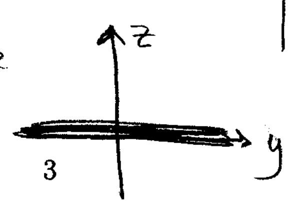
or b/c it has negative z-values.

(also it's clearly a plane and $f(x,y)=|x+y|$ isn't)

It also has $y=0$ trace



and $x=0$ trace



which are wrong.

5. Consider the lines $\vec{l}_1(t) = (6t+1, 3t-1, 2t+2)$ and $\vec{l}_2(t) = (t+3, \frac{1}{2}t+1, \frac{1}{3}t-1)$.

(a) (10 points) Show that $\vec{l}_1(t)$ and $\vec{l}_2(t)$ are parallel.

$\vec{m}_1 = (6, 3, 2)$, $\vec{m}_2 = (1, \frac{1}{2}, \frac{1}{3})$

$\vec{m}_1 = 6\,\vec{m}_2$ so the vectors are parallel hence the lines are.

(b) (10 points) Write the equation of the plane through these two lines. Your final answer must have the form $Ax + By + Cz = D$.

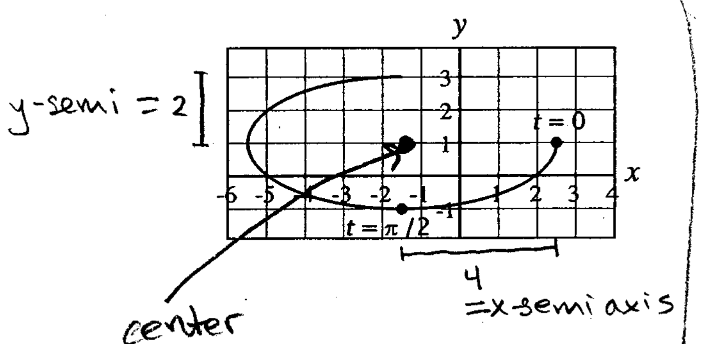
we need a point $\vec{x}_0$ on plane and $\vec{n} \perp$ to plane.
Since $\vec{l}_1(t)$ is on the plane, we can take $\vec{x}_0 = (1,-1,2)$
similarly using $\vec{l}_2(t)$ we could take $(3,1,-1)$ to be $\vec{x}_0$.

Since $\vec{m}_1$ and $\vec{m}_2$ are linearly dependent, we can't cross them to find $\vec{n}$. Instead take $(1,-1,2)-(3,1,-1)=\vec{v}$
$(-2,-2,3)=\vec{v}$

and then $\vec{n} = \vec{v} \times \vec{m}_1$ or use $\vec{n} = \vec{v} \times \vec{m}_2$

$\vec{n} = \vec{v} \times \vec{m}_1 = (-13, 22, 6)$, Plane: $(\vec{x}-\vec{x}_0)\cdot\vec{n} = 0 \longrightarrow -13x + 22y + 6z = -23$

6. (10 points) Write the parametrization for the portion of the ellipse pictured. Where the marked points correspond to the indicated values of t. If we didn't have to worry about orientation, the parametrization would be



$$\vec{f}(t) = (4\cos t - 3/2,\ 2\sin t + 1)$$

to make orientation counter-clockwise

$$\boxed{\vec{g}(t) = (4\cos t - 3/2,\ -2\sin t + 1) \quad 0 \le t \le 3\pi/2.}$$