

NAME: _____

YOUR GRADE IS BASED ON CORRECTNESS, COMPLETENESS, AND CLARITY ON EACH EXERCISE. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (15 pts.) Write the following system as a matrix, and row reduce (by hand) far enough to determine if the system is consistent. Do not completely solve the system.

$$\begin{array}{rccccrcr} x_1 & & & & - & 2x_4 & = & -3 \\ & 2x_2 & + & 2x_3 & & & = & 0 \\ & & & x_3 & + & 3x_4 & = & 1 \\ -2x_1 & + & 3x_2 & + & 2x_3 & + & x_4 & = & 5 \end{array}$$

2.) (15 pts.) Let $A = \begin{bmatrix} -1 & -3 & -3 \\ 2 & 6 & 1 \\ 3 & 8 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 & 1 \\ 1 & -5 & 6 \\ 7 & 3 & -3 \end{bmatrix}$.

a.) (3 pts.) Compute $A + B$. Show work for the top left entry.

b.) (4 pts.) Compute AB . Show work for the top left entry.

c.) (4 pts.) Compute A^{-1} . Show how to *set up* the appropriate algorithm. You can then use a calculator to jump to the end of the algorithm.

d.) (4 pts.) Change the last column of A so that the resulting matrix is *singular*.

3.) (15 pts.) Given the vectors $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$,

- a.) (5 pts.) Sketch \mathbf{v}_1 and \mathbf{v}_2 in the same x_1, x_2 -plane.
- b.) (5 pts.) Sketch $2\mathbf{v}_1 + \mathbf{v}_2$ and $3\mathbf{v}_1 - \mathbf{v}_2$. (Label all sketched vectors.)
- c.) (5 pts.) What is $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$?

4.) (15 pts.)

a.) (3 pts.) What is a system of equations of the form $A\mathbf{x} = \mathbf{0}$ called?

b.) (6 pts.) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & -1 & 3 \\ 1 & -2 & 1 & 1 \\ -2 & -1 & 1 & -3 \end{bmatrix}$. Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

c.) (6 pts.) Do the columns of A span \mathbb{R}^4 ? Explain.

5.) (15 pts.) Let $\mathbf{u} = \begin{bmatrix} 1 \\ -4 \\ 6 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 7 \\ 3 \\ -1 \end{bmatrix}$.

a.) Is $\{\mathbf{u}, \mathbf{v}\}$ linearly independent? Explain.

b.) Name a vector \mathbf{w} , not equal to either \mathbf{u} or \mathbf{v} , that is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$. Describe how you know \mathbf{w} is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$.

c.) Name a vector in \mathbb{R}^3 that is NOT in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$. Describe how you know this vector is not in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$.

6.) (15 pts.) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that $T(x_1, x_2) = (5x_1 - 4x_2, -2x_1 + 3x_2, x_1 - 3x_2)$.

a.) (5 pts.) Write the standard matrix for this linear transformation.

b.) (5 pts.) Is $T(\mathbf{x})$ one-to-one? Explain.

c.) (5 pts.) Is $T(\mathbf{x})$ onto? Explain.

7.) (10 pts.) In the downtown section of a certain city two sets of one-way streets intersect as shown in the figure below. The average hourly volume of traffic entering and leaving this section during rush hour is given in the diagram. Determine the amount of traffic between each of the four intersections. Then give specific amounts of traffic in the case that $x_4 = 200$.

